

## A Fuzzy AHP and TOPSIS for ELV Dismantling Selection

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*Abstract: In this paper, a new method for ELV dismantling selection is proposed which has purpose to determine the sequence of dismantling of ELV which arrive to dismantling center. Its solution is of the most importance for ELV waste management problem, as it influences the choice and efficiency of ELV waste management strategies in dismantling centers. It is realistically posed that the choice of locations depends on multiple, rather conflicting criteria. The criteria values can be either crisp or uncertain. The main contribution of this paper is the development of methodology for determining the dismantling order of coming ELV and/ or ELV which are already in the dismantling centers. The uncertain criteria values are described by linguistic expressions modeled by triangular fuzzy numbers. The extension of the fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is applied to rank the ELV which are in the dismantling center.*

*Keywords: End-of-Life Vehicles selection, dismantling centers, uncertainties, multi-criteria approaches, fuzzy sets*

### 1. INTRODUCTION

The prevention of wastage of vehicles, the improvement of vehicle dismantling and their recycling is another policy of Directive 2000/53/EC (EC (2000) Directive 2000/53/EC) which, among other, defines the obligations and rights of all partakers in this process: a country as a site owner, car-manufacturers, power and water producers, spare parts manufacturers, drivers, recycles, etc. The primal aim of such a policy is to achieve the economic success of End-of Life Vehicles (ELV) dismantling processes (target value of ELV recyclability level is about 95%).

In the past decade, recycling of ELV has arisen as a very important issue for car-manufactures worldwide, and the improvement of ELV recycling processes

has become one of their most important tasks. Moreover, problem of ELV recycling is not exclusively faced by industrialized countries. According to (Togawa, 2006), the implementation of strict product-oriented legislation (Life-Cycle- Assessment standard is associated with car-industry) will sooner or later become of dominant interest in developing countries.

Determining dismantling order of ELV in dismantling center is one of ELV waste management problems. The treated problem has become of special importance for developing countries, such as Serbia where ELV recycling process is in its infancy. The importance of the considered problem can be illustrated with the fact that productivity of dismantling centers depends in the first place on the chosen dismantling order of the ELV.

It is mentioned, that selection of

dismantling order of ELV is not a straight forward task. Many aspects, such as environmental features, social impact assessment, cost considerations, etc. must be accounted for, in order to point to an adequate management of recycling processes.

The considered problem is a group decision-making problem under multiple criteria, and it can be stated as multi-criteria optimization problem (MADM). The degree of uncertainty, the number of decision makers and nature of criteria those have to be taken into account in solving this problem. The different decision criteria may vary depending on the need of the organization and changing of environment. Estimation of criteria weights and uncertain criteria values cannot be performed with an exact numerical value. A more realistic approach may be to use linguistic assessments instead of numerical values. In other words, the all uncertainties which exist in considered problem can be described by linguistic variables (Zadeh, 1975). In this paper, modeling of these linguistic variables is based on the fuzzy set theory (Zadeh, 1975, Zimmermann, 1996, Pedrycy, and Gomide, 1998).

Fuzzy set theory can provide a valuable tool which copes with three major problematic areas of selection problems: imprecision, randomness and ambiguity. As far as imprecision is concerned it provides a powerful tool to weigh selection criteria importance. As far as ambiguity is concerned it copes better than other methods with the treatment of linguistic variables. Fuzzy logic enables us to emulate the human reasoning process and make a decision based on vague or imprecise data (Kaur, and Chakraborty, 2007).

The fuzzy set theory resembles human reasoning in its use of approximate information and uncertainty to generate decisions; based on natural languages; provide better communication between

experts and managers.

Optimal dismantling order of ELV in dismantling center can be obtained by applying some developed MADM techniques or by combining different MADM techniques. The widest appliance in multi-criteria problem for selecting of different items has AHP method and TOPSIS method or their combination (Ho, Xu, and Dey, 2009). In AHP method (Saaty, 1990) decision-making problem is hierarchy structured, the weights criteria and preference of alternatives under each treated criterion are assigned according to pairwise comparison matrix of considered factors. The elements of these matrixes are obtained upon the evaluation of decision makers. These matrixes represent input data for ranking and best alternative selection with respect to all treated criteria and their weights. TOPSIS (Hwang, and Yoon 1981) is based on the best alternative selection, which has the shortest distance from the positive-ideal alternative and the longest distance from the negative-ideal alternative. In the conventional forms of the AHP and TOPSIS methods, only crisp parameters values have been considered for supplier selection, which represents their basic defect.

In many papers, which can be found in the literature, MADM problems are solved by a proposed two-stage method. At the first stage, AHP or FAHP method is used to determine the weight of treated criteria and/or the weight of suppliers (Cebi and Bayraktar, 2003, Wang, et al, 2004, 2005, Percin, 2006, Chen, Lin, and Huang, 2006, Chan, and Kumar, 2007, Xia and Wu , 2007, Torfi, Farahani, and Rezapour, 2009). Secondly, some other methods are used in order to determine the best alternative with respect to all treated criteria, simultaneously, and their relative importance.

In this paper, we constructed: (1) a pairwise comparison matrix of relative importance of considered criteria, and (2) a pairwise comparison matrix of preference

of alternative. The elements of these matrixes are defined as: the relative importance/preference of criterion  $k/a$  over criterion. It appears that the weight determination of criteria is more reliable when using pairwise comparisons than obtaining them directly, because it is easier to make a comparison between two criteria than make an overall weight assignment. Also, the relative importance of each pair of treated criteria is described by linguistic expressions by more decision makers. The fuzzy rating of each decision maker is modeled by triangular fuzzy numbers. Weights vector is calculated by using the extent analysis method (Chang, 1996).

The criteria values can be crisp and uncertain. Normalization of crisp criteria is performed according to linear normalization procedure (Pomerol, and Barba, 2000).

A closeness coefficient according to determine rank of alternatives according to which we can determine is calculated by using procedure which is defined in conventional TOPSIS method.

The paper is organized as following. In Section 2 a multi-criteria approach for ranking of ELV in dismantling problem is given. In Section 3, synthetic extent analyses method for calculating the final priority weights of criteria and alternative preference is presented. In Section 4, principles of modified of TOPSIS method are presented.

## 2. MULTICRITERA APPROACH FOR SUPPLIER SELECTION

### 2.1 Basic assumptions

In fact, ELV for dismantling ranking and selection problem is a group multiple-criteria decision-making problem. Assumptions, under which a model for considered problem, are:

- Management Team defines the possible quantity of ELV which comes to dismantling center, unit/ time. The assessment of decision makers are is based on: (a) analysis of historical data, on the experience of other dismantling centers, (b) professional observation, etc. In practice different approaches are more often combined.

- Management Team defines the group of criteria according to which each ELV which is in the dismantling center and should be dismantled is being evaluated. The problem of selection of criteria according to which ELV for dismantling are evaluated and selected can be observed as an isolated problem. In this paper, we consider the following criteria: (1) age of vehicle, (2) the need for spare parts (spare parts), (3) possession of special tools for dismantling of ELVs, (4) condition of cars, (5) income / unit ELV, and (6) the level of pollution of water and soil by the ELV.

- To each defined criterion an organized pair (relative importance, value joined) is associated.

- Relative importance of treated criteria does not depend on ELV for dismantling, and is in most cases hardly changes. Generally, the relative importance of criteria is different and determined according to knowledge and experience of Management Team, and it can be stated as the following ways. Values of defined criteria are determined for each ELV for dismantling separately. In this problem, these values can be crisp and/or uncertain.

### 2.2 Notation

$a$ -alternative (ELV for dismantling),  
 $a=1,\dots,A$ ,  
 $k$ -criterion,  $k=1,\dots,K$ ,  
 $A$ -total number of ELV for dismantling,  
 $K'$ -total number of crisp criteria,  
 $K$ -total number of treated criteria,  
 $E$ -total number of decision makers who are

the members of the Management Team

$\tilde{W}_{kk}^e$  - a triangular fuzzy number

$\left(x; l_{kk}^e, m_{kk}^e, u_{kk}^e\right)$  describing the

fuzzy rating of relative importance of each pair of considered criteria of each decision maker,

$\tilde{W}_{kk}^+$  - a triangular fuzzy number

$\left(x; l_{kk}^+, m_{kk}^+, u_{kk}^+\right)$  describing the

relative importance of each pair of treated

criteria,  $(k=1, \dots, K)$ ,

$w_k$  - the relative importance of criterion  $k$ ,  $k=1, \dots, K$ ,

$f_{ak}$  - cardinal value of criterion  $k$  for alternative  $a$ ,  $k=1, \dots, K$ ;  $a=1, \dots, A$ ,

$f_{ak}^n$  - normalized value of  $f_{ak}$ ,  $k=1, \dots, K$ ;  $a=1, \dots, A$ ,

$\tilde{P}_{aa}^e$  - a triangular fuzzy number

$\left(y; l_{aa}^e, m_{aa}^e, u_{aa}^e\right)$  describing the

fuzzy rating of preference of each pair of alternative under criterion  $k$ ,

$k = K' + 1, \dots, K$  of each decision maker,

$\tilde{P}_{aa}^+$  - a triangular fuzzy number

$\left(y; l_{aa}^+, m_{aa}^+, u_{aa}^+\right)$  describing the

preference of each pair of treated alternative under uncertain criterion  $k$

criteria,  $k = K' + 1, \dots, K$ ,

$p_{ak}$  - the preference of alternative  $a$  under criterion  $k$ ,  $a=1, \dots, A$ ;  $k = K' + 1, \dots, K$ ,

$v_k^+$  - positive-ideal value of criterion  $k$ ,  $k=1, \dots, K$ ,

$v_k^-$  - negative-ideal value of criterion  $k$ ,  $k=1, \dots, K$

$c_a$  - a closeness coefficient for alternative  $a$ ,  $a=1, \dots, A$ .

### 3. MODELLING OF UNCERTAINTIES

All the criteria for evaluating ELV for dismantling usually do not have the same relative importance, and does not depend on ELV for dismantling. Also, it can be considered as unchangeable during the considered period of time. Determining the importance and alternative preferences under uncertain criteria involve a high degree subjective judgment and individual preferences of decision makers. We think that the judgment of each pair of treated criteria, regarding each pair of the considered alternatives best suits human-decision nature (analogously AHP method). In conventional AHP, the pairwise comparison is established using a standard integer scale [1-9]. Value 1 denotes that item  $i$  (criterion  $k$  and alternative  $a$ ) is important as criterion  $k$ , and value 9 denotes that item  $i$  is extremely more important/preferential than item  $i', i \neq i'$ .

Using the discrete scale of AHP is simple and easy but it is not sufficient to take into account the uncertainty associated with the mapping of one's perception to a number (Kwong, and Bai, 2003). Decision makers express their judgments far better by using linguistic expressions than by representing in terms of precise numbers. It feels more confident to give interval judgments than fixed value judgments.

In this paper, the fuzzy rating of each decision maker is described by linguistic expressions which can be represented as triangular fuzzy number

$\tilde{W}_{ii}^e = \left(x; l_{ii}^e, m_{ii}^e, u_{ii}^e\right)$  with the

lower and upper bounds  $l_{ii}^e, u_{ii}^e$  and

modal value  $m_{ii}^e$ , respectively. The

greater  $u_{ii}^e, -l_{ii}^e$ , the fuzzier the degree.

Values in the domain of these triangular fuzzy numbers belong to real set into interval [1-9]. Value in domain of each these five fuzzy numbers has the same meaning as value of standard scale which is given in conventional AHP.

If strong relative importance of criterion  $k'$  over criterion  $k$  holds, then pairwise comparison scale can be represented by the fuzzy number

$$\tilde{W}_{ii}' = \left( \tilde{W}_{ii}' \right)^{-1} = \left( \frac{1}{u_{ii}'} , \frac{1}{m_{ii}'} , \frac{1}{l_{ii}'} \right).$$

If  $i=i'$  then relative importance criterion  $k'$ /preferentmost alternative  $a'$  over criterion  $k'$ /prema alternativu  $a'$  is represented by single point 1 which is triangular fuzzy number (1,1,1).

The aggregated fuzzy rating of relative importance of each pair of considered criteria/preferentmost of each pair of treated alternatives must include the fuzzy rating of all decision makers. The aggregated fuzzy rating can be defined as:

$$l_{ii}' = \min_{e=1, \dots, E} l_{ii}^e,$$

$$m_{ii}' = \frac{1}{E} \cdot \sum_{e=1}^E m_{ii}^e, \quad u_{ii}' = \max_{e=1, \dots, E} u_{ii}^e,$$

The relative importance of each pair of considered criteria/preferentmost of each pair of treated alternatives is described by triangular fuzzy number

$$\tilde{W}_{ii}' = \left( l_{ii}', m_{ii}', u_{ii}' \right)$$

with the lower and upper bounds  $l_{ii}', u_{ii}'$  and modal value  $m_{ii}'$ , respectively.

In this paper, the fuzzy rating of each decision maker can be described by using a five linguistic expressions: *equally importance*, *moderately importance*,

*strongly importance*, *very strongly importance*, and *very strongly importance*. These linguistic expressions are modeled by triangular fuzzy numbers which are given in the following way:

*equally important/preference*  $\tilde{R}_E = (x; 1, 1, 2)$

*moderately important/preference*  $\tilde{R}_M = (x; 1.5, 3, 4.5)$

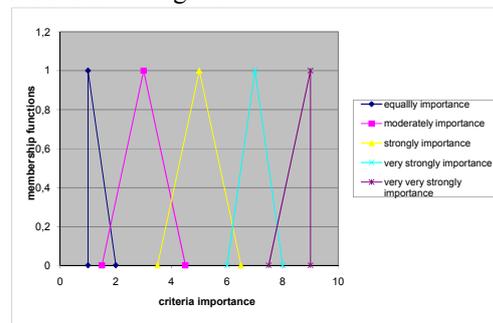
*strongly important/preference*  $\tilde{V}_S = (x; 3.5, 5, 6.5)$

*very strongly important/preference*  $\tilde{R}_{VS} = (x; 6, 7, 8)$

*very strongly important/preference*  $\tilde{R}_{VVS} = (x; 7.5, 9, 9)$

As one can see there are three basic terms low, medium and high importance. Two additional terms with the word *very* are obtained by moving strict boundaries. All five terms are given in Fig. 1.

These triangular fuzzy numbers, as the simplest shape of membership functions are shown in Fig. 1.



**Figure 1 – Five triangular fuzzy numbers that describe relative importance of criteria**

Weights vector of considered criteria and vector of preferentnosti alternative under uncertain criteria are calculated by

applying the concept of extent analysis (Chang, 1996).

#### 4. PRINCIPLES OF MODIFIED TOPSIS METHOD

TOPSIS method is based on choosing the best alternative, which has the shortest distance from the positive-ideal solution, and the longest distance from the negative-ideal solution (Hwang, and Yoon, 1981).

In this section, a systematic approach to extend the TOPSIS is proposed to solve the supplier selection problem under uncertainties.

The algorithm of the proposed method is realized in the following steps:

*Step 1.* Calculation of weights vector of considered criteria by applying procedure which is presented in Section 3.

*Step 2.* Calculation of preferentnosti alternativa under uncertain criterion  $k$ ,  $k = K' + 1, \dots, K$ .

*Step 3.* Calculation of normalized values for crisp criteria:

(a) benefit-type of criteria

$$f_{ak}^n = \frac{f_{ak}}{\sum_{a=1}^A f_{ak}}$$

(b) cost type-criteria

$$f_{ak}^n = 1 - \frac{f_{ak} - f^{\min}}{f^{\max}}$$

$$f^{\min} = \min_{a=1, \dots, A} f_{ak},$$

$$f^{\max} = \max_{a=1, \dots, A} f_{ak}$$

*Step 4.* Determining of positive-ideal solution,  $v_k^+$ , and negative-ideal solution,

$$v_k^-:$$

(a) for a crisp criterion criterion  $k$ ,  $k=1, \dots, K'$

$$v_k^+ = \max_{a=1, \dots, A} f_{ak}^n, \quad v_k^- = \min_{a=1, \dots, A} f_{ak}^n$$

(b) for uncertain criterion criterion  $k$ ,  $k = K' + 1, \dots, K$

$$v_k^+ = \max_{a=1, \dots, A} p_{ak}, \quad v_k^- = \min_{a=1, \dots, A} p_{ak}$$

*Step 5.* Calculate distance of each ELV for dismantling  $a$ ,  $a=1, \dots, A$ , from positive-

ideal solution,  $\tilde{d}_a^M$  and negative-ideal

solution,  $\tilde{d}_a^m$ , are calculated:

$$d_a^M = \sum_{k=1}^{K'} w_k \cdot |v_k^+ - f_{ak}^n| + \sum_{k=K'+1}^K w_k \cdot |v_k^+ - \tilde{p}_{ak}|$$

$$\tilde{d}_a^m = \sum_{k=1}^{K'} w_k \cdot |v_k^- - f_{ak}^n| + \sum_{k=K'+1}^K w_k \cdot |v_k^- - \tilde{p}_{ak}|$$

*Step 6.* A closeness coefficient,

$c_a$  ( $a = 1, \dots, A$ ) is obtained as:

$$c_a = \frac{d_a^m}{d_a^M + d_a^m}$$

#### 5. THE CONCLUSIONS

The experience of developed countries all over the world points out the necessity of developing recycling industry and integrating it with other industries with the aim of maintaining the market

competitiveness. This effect is reaching developing countries, and Serbia will be affected sooner or latter. Also, the demand for ELV waste management has a growing trend, which means that Serbia, having a car industry, should react promptly and undertake all necessary actions that would enable the development of ELV recycling processes.

It can be concluded that determining the optimal sequence of ELV dismantling which are located in dismantling center depends on many different criteria, such as: economic group criteria, social group criteria, environmental group criteria, etc. These criteria are very often in conflict. The considered criteria have a different relative importance and values criteria can be crisp or uncertain.

In this paper, weights vector and alternatives preferences vectors under uncertain criteria are obtained by applying extent analysis approach for the synthetic extent values of the pairwise comparison

for handling fuzzy AHP. TOPSIS method is used for ranking ELV in dismantling center.

It is shown that fuzzy sets are suitable for modeling of the uncertain input data in the considered ELV waste management problem that are subjectively estimated. The developed fuzzy models are flexible: (1) they include and operate with both precise and imprecise specific data, (2) all the changes, as the changes in the number of criteria or its relative importance, or number of possible locations and membership functions shape of fuzzy numbers can be easily incorporated into the model, and (3) fuzzy model could be modified for solving different waste management problems.

Further research will be carried out primarily in the analysis of fuzzy developed model sensitivity to different shapes of fuzzy terms membership functions which appear in the model.

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