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FAZZY RATING OF CRITERIA WEIGHTS OF MANAGEMENT PROBLEMS IN ELV RECYCLING RESEARCH AREA

Abstract: *Determining criteria weights that enable rating the alternatives in different ELV recycling management problems can be observed as an independent task. This paper gives a short summary of different approaches which can be found in the literature and which are used to calculate criteria weights. The principles of fuzzification of Delfi method are presented.*
Keywords: *ELV recycling problem, multi-criteria optimization, criteria weights, fuzzy sets*

1. INTRODUCTION

In the past decade, recycling of ELV has arisen as a very important task for car-manufacturing industry worldwide. It can be stated as multi-criteria optimization task under group decision making process. Determining criteria weights is a complex task and can be observed as an independent task. Generally speaking, the criteria importance or criteria weight can be defined in 3 different ways [1]: normalized, with a relative importance of criteria and using linguistic expressions.

When criteria weights are given in a normalized form, then their values are described by a set of real numbers that belong to the interval [0-1] and their i njihov sum is equal to one. This approach to criteria weights definition is simple and easy to understand. Although, this approach demands from the decision-maker to map their evaluation to a numerical scale of measure. It is more natural to use linguistic expressions to state his evaluation. Furthermore, if the decision problem consists of a large number of criteria, it is very difficult for a decision-maker to focus on all criteria in the same time.

Based on the above stated drawbacks of the first way of criteria weights evaluation, authors consider that the other two ways of defining and evaluating criteria weights in recycling of ELV problem are more adequate.

Using a numerical scale, as it is defined in the conventional AHP method is simple and understandable from the decision-maker's point of view. It is realistic to assume that decision-makers express their evaluations easier using linguistic expressions rather than using numbers. The development of mathematics theories such as: probability theory, fuzzy set theory, rough set theory enabled to adequately describe linguistic expressions mathematically. Many papers showed that the fuzzy set theory has several advantages in describing linguistic expressions over other two mentioned theories. Advantages of the fuzzy approach in describing linguistic expresions can be expressed as follows: (1) calculating the distribution of a random variable requires a lot of data records, (2) combination of different uncertainties leads to a complex probability distribution, which requires complex mathematical expressions.

Fuzzy approach in the treatment of uncertainty enabled the elimination of uncertainty and inaccuracy due to lack of good data records. This approach to describing uncertainty should be applied in the following cases: (1) where the conditions are constantly changing so it is impossible to describe the observed variable with a random variable, and (2) where there is not enough relevant data records that are necessary for statistical analysis.

Linguistic expressions can be adequately described using fuzzy set theory [2]. Membership function of a fuzzy set is a convex function that has a shape of a triangle, trapeziod or a shape of Gaussian curve. In many cases, per example in the problem of modelling criteria weights, the shape of the membership function does not significantly influence the final result.

The fuzzy set theory can provide a valuable tool which copes with three major problematic areas of supplier selection: imprecision, randomness and ambiguity. As far as imprecision is concerned, it provides a powerful tool to weigh selection criteria importance. As far as ambiguity is concerned, it copes better than other methods with the treatment of linguistic variables. Fuzzy logics enables us to emulate the human reasoning process and make a decision based on vague or imprecise data [3].

The fuzzy set theory resembles human reasoning in its use of approximate information and uncertainty to generate decisions. We can come to a conclusion that the fuzzy approach in treating uncertainties in real-word applications has numerous advantages when compared to other approaches such as applying probability theory, applying rough set theory, etc. These advantages of fuzzy system models could be expressed as follows [4]: conceptually easy to understand; flexible; capture most nonlinear functions of arbitrary complexity; tolerant of imprecise data; built on the expertise of experts; could be blended with conventional control techniques; based on natural languages; provide better communication between experts and managers.

The paper is organized as follows: Section 2 describes the method for determining the criteria weights when they are given with a matrix of comparison pairs of the relative ratio of criteria importance, Section 3 describes the method for evaluating the criteria weights hen they are described using linguistic expressions and Section 4 gives the conclusions and further research directions.

2. RELATIVE RATIO OF CRITERIA IMPORTANCE

In the conventional Analytic hierarchy process (AHP) [5] an assumption is introduced, that the perception of decision-maker about the relative ratio of criteria importance can be mapped to a numerical scale, whose values belong to an interval [1-9]. Value 1 denotes that criterion k is as important as criterion k', and value 9 denotes that criterion k is much more important than an assumption is introduced that the decision-makers make their decisions by consensus. Using eigenvector methods[5], which is considered a natural measure of inconsistency, an evaluation of criteria weights can be done, even when there are errors in the assessment. "Errors" in the assessment of a decision-maker can be measured using a consistency index, which is denoted by C.I. It can be concluded that a pairwise comparison matrix of the relative importance is consistent if $\mathrm{C.I.} \leq 0.01$. If this condition is not met, i.e. if the matrix is not consistent, then a correction of decision-maker's evaluation needs to be performed.

In many papers which consider various management problems that belong to different domains, criteria weights are determined using a procedure that is developed in the conventional AHP method. ([6],[7],[8],[9],[10]).

In literature it is possible to find a large number of papers that describe the relative ratio of criteria importance using linguistic expressions that are modeled by fuzzy numbers. Many papers in literature ([11],[12],[13],[14]) evaluate the criteria weights using the extent analysis method by Chang ([15]. These papers make an assumption that the relative importance is described using a predefined set of linguistic expressions that are modeled by fuzzy numbers. The domains of these fuzzy numbers are defined on a set of real numbers that belong to different intervals. In [12] the problem is adressed as a fuzzy multi-criteria group decision problem. Authors developed a method by which the opinion of all decision-makers is aggregated into a single triangular fuzzy number in the following manner: lower limit, or upper limit of the fuzzy number is calculated as the minimum, or maximum value of all fuzzy numbers that describe the evaluation of the decision-maker. Modal value of the fuzzy number which describes the aggregated oppinion of all decision-makers is calculated as the mean value of all the modal values of fuzzy numbers that represent fuzzy evaluations of decision-makers. Weight vector of the observed criteria is calculated using the method described in [15].

3. CRITERIA WEIGHTS DESCRIBED USING LINGUISTIC EXPRESSIONS

Many papers that can be found in literature state that it is possible to describe criteria weights using linguistic expressions that are modeled by fuzzy numbers. A short overview of these papers is given in the sequel.

In [16] the criteria weights are described by trapezoidal fuzzy numbers. They are input data of weighted normalized decision matrix. Mahdavi i dr. [17] are supposed that each decision maker assesses criteria weights by using ahead defined linguistic expressions. These linguistic expressions are modelled by trapezoidal fuzzy numbers. The domains of these fuzzy numbers are defined on the set of real numbers into interval [0-1]. In [18] the considered problem is stated as group decision making problem. The criteria weights are evaluated by each decision maker. The decision makers used ahead defined linguistic expressions. The nine linguistic expressionas are proposed by authors. These linguistic expressions are modelled by trapezoidal fuzzy numbers which are defined into interval [0-1]. The aggregated fuzzy rating of criterion weight are calculated by mean value method. In [19] every decision-maker directly estimates the importance of observed criteria using seven predefined linguistic expressions. These linguistic expressions are modeled by triangular fuzzy numbers. The domains of the fuzzy numbers are defined in interval [0-1]. Value 0 means that the criterion has the lowest priority and value 1 means that it has the top priority. Criterion weight k, k=1,..,K is calculated as the mean value of evaluations of all decision-makers. In [20] the observed problem is set as a group decision-making task respecting several criteria and their weights. Every decision-maker describes the criteria weights using intuitive fuzzy sets. Aggregated criteria weight is determined using the $IFWA_{\lambda}$ operator [21]. In [22] relative criteria weights are described by discrete fuzzy numbers, whose domains are defined on a real number set, in interval [1-9]. Value 1 denotes that the criterion has the lowest priority and value 9 denotes that it has top priority. The membership value for every domain value is calculated based on the subjective assessment of the decision-maker. The procedure for transforming of relative importance of criteria in degree belief, B_k is proposed. The degree belief B_k is belief measure of decision maker that criterion k, k=1,..,K is equal or more imortant then all rest considered criteria. The belief measure is performed by using comparison method of discrete fuzzy numbers. In [23] the authors have suggested that criteria weights are determined directly and that doctors can use one of four linguistic expressions that are modeled by fuzzy numbers. Fuzzy numbers are is this case defined on a real numbers set, belonging to an interval [1-9] (analogous to the conventional AHP method). The criteria weights are calculated by metod for comparison of discrete fuzzy numbers. In [24] the criteria weights are evaluated by usinf five linguistic expressions which are modelled by triangular fuzzy numbers. Domein of each fuzzy number is defined on the set of real numbers into interval [0-10]. In [25] fuzzy Delfi method is developed. The fuzzified Delphi method can be used for criteria weights rating in different management problems.

4. COMBINED METHODS FOR DETERMINING CRITERIA WEIGHTS

Many papers suggest that criteria weights should be determined by combining the two previously described approaches.

In [26] it was assumed that pairwise comparison matrix of criteria weights are constructed. In this paper a new procedure for evaluation of criteria weights is developed. This procedure is conducted through the following steps: (1) decision-maker evaluates pairwise relative importance using the standard scale of measure; before calculating the criteria priority vector, elements of the pairwise comparison matrix are normalized using the linear normalization procedure, (2) evaluation consistency index is determined (analogous to the AHP method); in case consistency is not reached, the decision-maker has to start over and repeat the evaluations [27], (3) elements of the pairwise comparison matrix are tranformed to fuzzy numbers; authors suggest five triangular fuzzy numbers for relative importance evaluation. These fuzzy numbers are defined in interval [0-1]. Value 0 denotes that criterion has the lowest priority and value 1 denotes that it has the top priority, (4) criteria weights k, k=1,..,K are calculated as the mean value of row elements k, k=1,..,K of the transformed pairwise comparison matrix of the relative importance. In ([28],[29],[30]) assumptions are made that the pairwise relative importance of observed criteria can be described using three linguistic expressions. These linguistic expressions are modelled by triangular fuzzy numbers which are defined on standard measurement scale measures. By using the equal possibilities concept [1], for each level of membership function the criteria weight vector is calculated. The criterion weight is described by discrete fuzzy number.

5. DISCUSSION AND FURTHER RESEARCH DIRECTIONS

As it was shown in this paper, it is possible to find different approaches to determining criteria weights in different management problems in literature. Many authors consider that the evaluation of pairwise importance is more adequate way to determine vector of criteria weights because this approach is closer to human thinking process. This attitude is absolutely acceptable if the management problems consists of a large number of criteria. On the other hand, if decision-makers define a relatively small number of criteria, then a direct evaluation of criteria importance is a quite acceptable way to evaluate criteria weights.

In management problems, such as recycling of ELV problem which exists in the real world, it is more realistic to set up a group decision-making task in the presence of uncertainty. Decision-makers express their oppinions and attitudes best if they use linguistic expressions. Linguistic expressions modeling is based on the fuzzy set theory. It makes sense to apply this approach in those cases where there is a source of incertainty and imprecision of any kind.

In the mentioned papers that can be found in literature, several different methods of aggregation of fuzzy evaluations of the decision-makers are proposed. Several authors consider that the best way to synthesize different oppinions of decision-makers into a unique evaluation is to apply the Delfi method. This method has a wide range of applications for solving different management problems.

Fuzzified Delfi method can be applied when both ways for definition and calculation of criteria weights are used. In case criteria weights are defined using the pairwise comparison matrix of the relative importance, every element of this matrix is evaluated using the fuzzified Delfi method. Vector of criteria weights can generally be obtained in three different ways: (1) applying the extent analysis method [15], (2) using the equal possibilities principle [1], and (3) element of the weights vector (criteria weights k, k=1,.,K) is calculated as the mean value of aggregated evaluation of row k, k=1,..,K obtained by applying the fuzzified Delfi method. If the direct evaluation method is used, criteria weights are obtained using the fuzzified Delfi method.

The fuzzification of Delfi method can be briefly expressed in the following manner: (1) in the first step decision-makers express their evaluations using linguistic expressions that are modeled by fuzzy numbers; based on these values the aggregated evaluation of fuzzy relative importance of criteria and/or criteria weights is calculated. This value is obtained as the mean value of all evaluations. Then another quantity is calculated - the distance of obtained fuzzy number from all fuzzy numbers which have intersection different than zero. The value of this distance is crisp. Decision-makers receive in written form the information about distance values based on the evaluations they perform in the second round; (2) in the second step, decision-makers need to adjust their evaluations based on analysis results obtained in the first step; the analysis of results obtained in this step is performed in the same way as it was already described. It is assumed that the consensus is reached if there is no significant difference between weights values in the first and second step. If the consensus is not reached, then the procedure described in Step 2 is repeated until reaching a consensus.

On the basis of all the stated, it can be concluded that the problem of rating criteria weights is very complex and can be observed as an independent task. The solution of this problem is propagated through the entire management problem and influences the solution and efficacy of the problem solving itself. This fact illustrates pretty well the significance of the considered problem.

The author suggests a modification of Delfi method in which fuzzy evaluation of decision-makers exist as input data. Basic principles of Delfi method are summarized in Section 5. The author considers that the combination of fuzzy Delfi method with existing ways for determining criteria weights can greatly improve the accuracy of criteria weights evaluation.

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