Branka Dimitrijević[1)]
Dražen Popović[1)]
Vladimir Simić[1)]

1) University of Belgrade-Faculty of Transport and Traffic Engineering, mail: {brankad, d.popovic, vsima}@sf.bg.ac.rs

UNCERTAINTY OF MINIMAL SAFETY DISTANCE IN DANGEROS GOODS FACILITIES LOCATION

Abstract: *In this paper we try to give an answer on the following question. Is it better, from the practical point of view, to respect uncertainty of minimal safety distance while solving problem for dangerous goods facilities location, or to solve it without any consideration of uncertainty? We used formulation of Anti-Covering Location Problem (ACLP) to describe the problem and then we analyzed effects of randomly generated distance of spreading undesirable effects on optimal solutions of ACLP, based on prescribed value for minimal safety distance between network nodes from one side, and on solutions of ACLP obtained for the case when spreading effects are considered as stochastic, and fuzzy value, from the other side.*
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1. INTRODUCTION

Numerous goods which are widely used and provide a utility to the user are known as "undesirable" or "obnoxious". Those goods may generate different undesirable effects which can be felt over a certain geographical space, and making decisions about spatial position of its storing facilities is crucial when it comes to minimize the environmental risks. Storing of dangerous goods like explosives, flammable materials and compressed gasses is characterized by the opportunity to transfer undesirable effects to the objects in the neighborhood thus causing destruction, serious damage and fire in these areas. Those effects spread spherically from the source, reaching surroundings within a certain radius known as minimal safety distance. The minimal safety distance usually depends on the quantity and characteristics of the activated material, as well as other relevant characteristics (building construction, the mutual spatial position of the donor and acceptor objects...). Furthermore, minimal safety distance is usually given as a constant prescribed value, defined by certain norms and regulations, even when spreading of undesirable effects (explosion, fire, compression, toxicity...) is uncertain and stochastic in nature.

The Anti-Covering Location Problem (ACLP) is a member of an important class of spatial optimization problems. Assume that there is a set of potential location sites with a distance or time measure associated with travel from one to all the other sites. Also, to each site is assigned positive weight relating the potential use of that location. The ACLP is to find the maximally weighted set of location sites such that no two selected sites are within a specified distance or time standard of each other.

When network is defined with the set of sites representing nodes, and the set of arcs representing pairs of nodes which are within the minimum separation distance or time standard, then the ACLP is equivalent to the Maximum Independent Set Problem (Node/Vertex Packing Problem). Further, this problem, and subsequently the ACLP, is closely related to the Maximum Clique Problem and Vertex Cover Problem. This indicates that a substantial body of research has been devoted to the ACLP and related problems [1,2]. Since the maximum independent set problem is NP-complete, the ACLP is also NP-complete combinatorial optimization problems.

A wide variety of particular applications of the ACLP and related problems can be found in literature [1,2]. Location of undesirable or obnoxious facilities (warehouses for dangerous goods, polluting plants, military defense location, as well as noise, odor or heat emitters) represents one important class of ACLP. As it is mentioned, ACLP is equivalent to the Node/Vertex Packing Problem, and than can be used for solving problems related to disposition of dangerous goods in transportation units and storages.

Having that in mind, we used formulation of ACLP to describe the problem in which we analyzed effects of randomly generated distance of spreading undesirable effects on optimal solutions of ACLP, based on prescribed value for minimal safety distance between network nodes from one side, and on solutions of ACLP obtained for the case when spreading effects are considered as stochastic, and fuzzy value, from the other side. In this way, this paper extends one part of research conducted in [3].

2. PROBLEM FORMULATION

There are few mathematical formulations of the ACLP proposed in the literature [1,2].

Let us introduce binary variable x_i defined in the following way:

$$x_i = \begin{cases} 1, & \text{if a node } i \text{ is chosen to be a facility location} \\ 0, & \text{otherwise} \end{cases}$$

Let us introduce the following notation:

$N = \{1, ..., i, j, ..., n\}$ – index set for potential facility sites;

v_i – node weight (site capacity restriction associated with the use of location i), vi>0;

c_{ij} – the shortest distance between node i and node j (in this case Euclidean distance);

R – pre-specified minimal safety distance (no two facilities may be closer than R);

$\Pi(i)=\{j|c_{ij}\leq R,\ i\neq j\}$ – nodes that are on distance less or equal to R, excluding particular node i;

M – sufficiently large positive number.

The ACLP was first defined and formulated as an integer programming problem [1] as follows:

$$\text{Max } Z = \sum_{i\in N} v_i x_i \tag{1}$$

$$Mx_i + \sum_{j\in\Pi(i)} x_j \leq M, \ \forall i \in N \tag{2}$$

$$x_i \in \{0,1\}, \ \forall i \in N \tag{3}$$

The objective function of the ACLP (1) that should be maximized represents the total weighted selection of the facility location sites. No two facility sites within the pre-specified minimum distance of each other are at the same time incorporated in the solution (Constraint (2)). If node i is selected for facility placement (i.e. x_i=1), then the term Mx_i equals the right hand side term M, and forces $\sum_{j\in\Pi(i)} x_j = 0$. Constraint (3) describes problem variables.

3. UNCERTAINTY IN ACLP

Stochastic approach to uncertainty

Bearing in mind previously mentioned fact that spreading of undesirable effects is usually uncertain, and stochastic in nature, usually normally distributed, it has sense to consider minimal safety distance as a normally distributed stochastic variable $X_R \sim N(\mu,\sigma)$.

Let $P(X_R<R)=\varepsilon$ be the probability that ε "undesirable effects" has spread under prescribed minimal safety distance R, (Fig.1), and let $K_V=\sigma/\mu$, be the variation of normal distribution with parameters μ,σ. When ε, and K_V are known, it is possible to obtain parameters μ and σ of normal distribution of stochastic variable X_R. Note also that the probability $P(X_R<R)$ denotes "protection factor" for minimal safety distance.

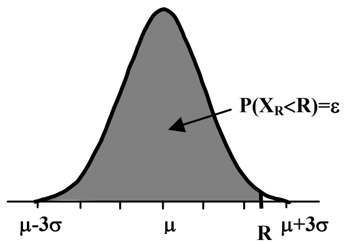

Figure 1. $P(X_R<R)=\varepsilon$

In addition, let $p_{ij}=P(X_R<c_{ij})$ be the probability that X_R takes value lesser then distance between arbitrary pair of nodes $i,j \in N$ (Fig. 2), where c_{ij} represents Euclidean distance between nodes.

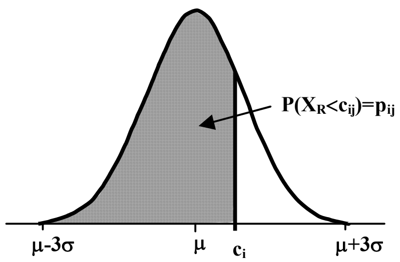

Figure 2. $P(X_R<R)=\varepsilon$

Based on above mentioned consideration, regarding to previously introduced notation, under assumption that minimal safety distance is stochastic variable, it makes sense following stochastic formulation of ACLP:

$$\text{Max } Z_S = Z \cdot P \tag{4}$$

$$\text{Max } Z = \sum_{i \in N} v_i x_i \tag{5}$$

$$P = \prod_{i,j \in N} p_{ij}, \; i = \overline{1, n-1}; \; j = \overline{i+1, n}; \quad i \neq j; x_i = x_j = 1 \tag{6}$$

$$P=1, \text{ iff } x_i=1 \wedge x_j=0, j=1, 2,\dots, i-1, i+1,\dots, n \tag{7}$$

$$x_i \in \{0,1\}, \; \forall i \in N \tag{8}$$

In this case optimality criterion (4) is maximization of ACLP solution quality measure expressed as a product of the quantity of goods stored (5), and the probability P that distance between all pairs of nodes in the solution (decision variables whose value is equal to 1) greater than the value of stochastic variable X_R (6). Quantity of goods stored Z (5), corresponding to maximal value of Z_S (4) is the best solution for the stochastic ACLP defined by (4)-(8). However, it should be emphasized that the best solution obtained by solving (4)-(8), among the solution quantity performance Z is described by the protection factor P of the solution as well. Note that P=1 when solution vector consists of only one node.

Fuzzy approach to uncertainty

Let $\tilde{R}$ be fuzzy number representing minimal safety distance. Authors analyzed numerous shapes of triangular fuzzy numbers, and accepted form is shown in Fig. 3. Obviously, shape of proposed fuzzy number is based on parameters of previously mentioned parameters of stochastic variable. The main idea was to express uncertain spreading of undesirable effects trough fuzzy, instead stochastic variables and compare similarities and differences between those two approaches through simulation process. To do that, it was necessary to compare fuzzy number

$\widetilde{R}$, and "crisp" Euclidean distances c_{ij} between pair of nodes $i,j \in N$.

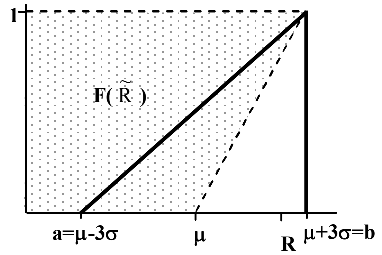

Figure 3. Possible shape of fuzzy number representing minimal safety distance and value of ordering function

In this research comparison of fuzzy and crisp numbers was based on Yager's method [4]. Yager's method requires determination of ordering function F, which transforms fuzzy number into crisp value in following way:

$$F(\widetilde{R}) = \int_0^{\alpha_{max}} M(\widetilde{R}_\alpha)\, d\alpha \tag{9}$$

Here, $M(\widetilde{R}_\alpha)$ denotes average membership function ($\widetilde{R}_\alpha = \{x | x \in [a,b], \widetilde{R}(x) \geq \alpha\}$) of fuzzy number $\widetilde{R}$, where $\alpha_{max}=1$, because fuzzy number is normalized. Value of $F(\widetilde{R})$ is shown in Fig.3, covering shaded area.

Since, minimal safety distance is now transformed into crisp number by using (9), it is possible to solve ACLP by using formulation (1)-(3), where instead $\Pi(i)=\{j|c_{ij}\leq R\}$, should be used $\Pi(i)=\{j|c_{ij}\leq F(\widetilde{R})\}$.

4. SIMULATION ANALYSIS OF ACLP

Simulation analysis of uncertainty in ACLP is realized on 4 groups of 240 randomly generated problems. The first group consists of 60 problems with 15 nodes of equal weights. Second group consists of 60 problems with 15 nodes of weights varying between equal to 10 times greater. Third group consists of 60 problems with 18 nodes of equal weights, and fourth group consists of 60 problems with 18 nodes having weights varying between equal to 10 times greater. Each group of 60 problems has divided into 3 groups of different "density" of network (different average distance between nodes). Size of test problems was limited by the possibility to solve ACLP with stochastic approach to minimal safety distance in reasonable time.

All 240 problems were solved optimally with constant minimal safety distance R=60, by using formulation (1)-(3), and then by using stochastic approach and fuzzy approach both described in previous section. Each problem analyzed by stochastic approach was solved for 16 combinations of values of ε, and K_V. Combinations analyzed were: $\varepsilon=0.997$, $\varepsilon=0.95$, $\varepsilon=0.90$, $\varepsilon=0.70$, and $K_V=1\%$, $K_V=5\%$, $K_V=10\%$ and $K_V=20\%$. Similarly, in case of fuzzy approach, 12 crisp values for minimal safety distance were defuzzyfied from the same 12 combinations of values for ε, and K_V. Hence, total number of solutions analyzed by simulation was 7920.

The optimal solutions for the selected test problems were solved using the CPLEX 12, while obtaining solutions for stochastic and fuzzy approach, as well as simulation process were realized in the Python 2.6. Simulation analyses was realized in a following way: one of the sites in the obtained solution is randomly selected as the location where the accident happened; range of realized accident is obtained from normal distribution with μ, and σ, calculated in according with ε, and K_V; previous procedure is repeated 10000 times for every solution. When ever accident was happened, following statistics were gathered: value of the objective function, number of simulations with accident that affected neighbor objects;

number of neighbor objects affected in solution, and total weight of neighbor objects affected in solution.

Actually, main idea of simulation experiment was to analyze performances of solutions obtained trough mentioned three approaches as it is shown in Fig. 4. Namely, when solutions obtained trough different approaches differ, as in Fig. 4, total number and weights of objects affected will also differ, and this may be used as solution approaches performance measure.

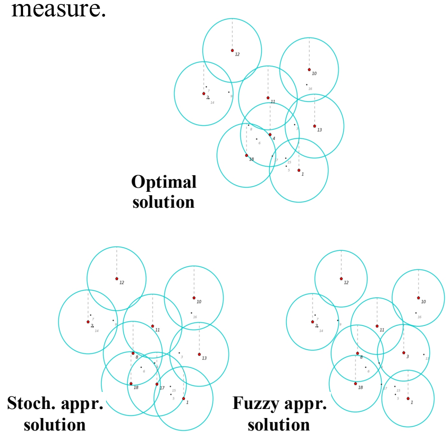

Figure 4. Different approaches and different solutions for the same problem

Table 1 shows simulation results related to the third group of 60 problems with 18 nodes equal weight. Results obtained for all other groups are very similar and thus we chose this one as a representative.

In research presented in paper [4] we did simulation in a bit different way, smaller number of times, also for smaller size problems, and just gathered overall statistics about number of affected objects when minimal safety distance is prescribed value and when it is defined respecting uncertainty. It was obvious that respecting nature of the process of spreading undesirable effects, which means respecting uncertainty in ACLP, has significant impact on solving this special class of location problem. But observing results separately some more conclusions appeared.

ε	K_V	R=const.		Stochastic R		Fuzzy R	
		ANS*	ANA*	ANS*	ANA*	ANS*	ANA*
0.997	0.01	0.02	0.00	0.01	0.00	0.10	0.01
	0.05	0.06	0.00	0.28	0.02	0.4	0.02
	0.1	0.10	0.01	0.45	0.03	0.54	0.03
	0.2	0.13	0.01	0.62	0.04	0.69	0.04
0.97	0.01	0.07	0.00	0.03	0.00	0.13	0.01
	0.05	0.21	0.01	0.18	0.01	0.53	0.03
	0.1	0.33	0.02	0.61	0.04	0.67	0.04
	0.2	0.49	0.03	0.83	0.05	1.00	0.06
0.95	0.01	0.14	0.01	0.05	0.00	0.13	0.01
	0.05	0.39	0.02	0.22	0.01	0.51	0.03
	0.1	0.57	0.03	0.43	0.02	0.70	0.04
	0.2	0.90	0.05	0.89	0.05	1.17	0.07
0.9	0.01	0.29	0.02	0.12	0.01	0.25	0.01
	0.05	0.84	0.05	0.52	0.03	0.34	0.02
	0.1	1.29	0.07	0.81	0.05	0.46	0.03
	0.2	2.03	0.13	1.10	0.07	1.17	0.07
0.7	0.01	1.14	0.06	0.1	0.01	0.07	0.00
	0.05	3.04	0.17	0.42	0.02	0.4	0.02
	0.1	5.15	0.31	0.68	0.04	0.68	0.04
	0.2	8.60	0.57	1.04	0.06	1.28	0.08

ANS*-Average number of simulations with the impact on adjacent objects
ANA*-Average number of affected objects

Table 1- Simulation statistics

For a very large value of "protection factor" (ε=0.997), regardless of the value of K_V, average number of affected objects in optimal solutions was smaller than in solutions obtained respecting uncertainty. For ε=0.97 average number of affected objects in all three types of solutions is almost the same. In all other cases (ε=0.95, ε=0.90, ε=0.70), especially for K_V=0.1, K_V=0.2, number of affected objects in optimal solutions was bigger than in solutions obtained respecting uncertainty.

Differences in solutions and simulation effects highlight conclusion that it is better from practical point of view to respect uncertainty while solving this class of location problem than to solve it without any consideration of uncertainty. This is especially important in situations where the location of hazardous materials (such as transport) can't be realized with great value for the "protection factor" ε. In that case (un)respecting uncertainty has significant impact on the quality ("safety") of the solution. Pre-specified minimal safety distance, if it is used for problem solving, should be determined by using the "protection factor" ε>0.97.

Having in mind that solving problem defined by formulation (4)-(8) is not an easy task for the problems of larger dimensions, we tried to describe minimal safety distance as a fuzzy number and compare results obtained. As it is shown in Table 1, simulation results obtained for proposed fuzzy approach are partially consistent with the results obtained for stochastic approach. Future research requires additional experiments with other forms of fuzzy numbers that will better describe the stochastic nature of minimal safety distance and allow easier problem solving.

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