# A NEW METHOD FOR CONSTRUCTING CONFIDENCE INTERVAL FOR CPM BASED ON FUZZY DATA

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***Abstract:*** *A measurement control system ensures that measuring equipment and measurement processes are fit for their intended use and its importance in achieving product quality objectives. In most real life applications, the observations are fuzzy. In some cases specification limits (SLs) are not precise numbers and they are expressed in fuzzy terms, so that the classical capability indices could not be applied. In this paper we obtain $100(1-\alpha)\%$ fuzzy confidence interval for $\tilde{C}pm$ fuzzy process capability index, where instead of precise quality we have two membership functions for specification limits.*

***Keywords:*** *Dp,q-distance; Fuzzy set; Membership function; Process capability index; Triangular fuzzy number; Fuzzy random variable.*

## 1. INTRODUCTION

Statistical techniques can be helpful throughout the product cycle, including activities prior to manufacturing, in quantifying process variability, in analyzing this variability relative to product requirements or specifications, and in assisting development and manufacturing in eliminating or greatly reducing this variability. This general activity is called process capability analysis. Process capability refers to the uniformity of process. Obviously, the variability in the process is a measure of the uniformity of output. There may not exist a definition of the ‘‘process capability’’ but in high probability the (real valued) quality characteristic X of the produced items lies between some lower and upper specification limits LSL and USL (or tolerance interval limits). Therefore the idea of process capability implies that the fraction p of produced nonconforming items should be small if the process is said to be capable. In the traditional quality management, the most commonly used capability indices like Cp, Cpk and Cpm are used to indicate process capability . An underlying assumption is that output process measurements are distributed as normal random variables. Experience shows that the normality assumption is often not met in real world. Application and observations usually contain fuzziness owing imprecise measurements or described by linguistic variables, For instance, the water level of a river cannot be measured in an exact way because of the fluctuation. Therefore we can just say that the water level is ‘‘around 32 m” and the phrase ‘‘around 32 m” can be regarded as the fuzzy number $\tilde{32}$. Similarly, temperature in a room is also unable to be measured precisely because of the same reason. Therefore, the fuzzy sets theory is found to be an appropriate tool in modeling the imprecise data. In some cases specification limits (SLs) are not precise numbers and they are expressed in fuzzy terms, so that the classical capability indices could not be applied. A. parchami et al [2] obtained fuzzy confidence interval for fuzzy process capability (C̃p) when SLs are imprecise. Chen, Lai and Nien [1] used the fuzzy analytic method concerning process capability index Cpm and calculate C̃pm for fuzzy observations. Perakis and Xekalaki [3] constructed confidence interval for the index Cpm with crisp data. In this paper, we introduce a new fuzzy PCI (C̃pm) when SLs, target and observations are triangular fuzzy number and introduced a 100(1− $\alpha$)% confidence interval for C̃pm fuzzy process capability index, when the engineering specification limits are triangular fuzzy numbers.

The organization of this paper is as follows. In section 2 we recall some notions of fuzzy numbers used in this paper. Section 3 contains the traditional definitions of process capability indices. Sections 4 and 5 assign to the presentation of point estimation for Cpm based on fuzzy data and a ranking function for compare between fuzzy numbers. We introduce a confidence interval for C̃pm with using fuzzy observation, fuzzy target and fuzzy specification limits in section 6. At last present a numerical example.

## 2. PRELIMINARIES

Let R be the set of real numbers. Set:

$F(R)=\{A \mid A: R\to[0,1], A is a continous function\}$,

$F_T(R)=\{T(a,b,c) \mid a,b,c\in R, a\le b\le c\}$,

where

$$T(a,b,c)=\begin{cases}(x-a)/(b-a) & if\ a\le x\le b,\\ (c-x)/(c-b) & if\ b\le x\le c,\\ 0 & elsewhere\end{cases} \quad (2.1)$$

Any $A\in F(R)$ is called a fuzzy set on R and any $T(a,b,c)\in F_T(R)$ is called a triangular fuzzy number.

***Definion 2.1.***

i) Let $T(a,b,c)\in F_T(R), k\in R, k\ge 0$. Define the operation $\otimes$ on $F_T(R)$ as follows

$$k\otimes T(a,b,c)=T(a,b,c)\otimes k=T(ka,kb,kc), \quad (2.2)$$

called the multipilication of $T(a,b,c)$ by $k$.

ii) For $\alpha\in[0,1]$, the $\alpha$ -cut of $T(a,b,c)$ is defined by

$$(T(a,b,c))_\alpha=\{x\in R \mid T(a,b,c)\ge\alpha\}.$$

***Definion 2.2.***

The Dp,q-distance, indexed by parameter 1< p < 1, 0 ≤q ≤ 1, between two fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ is a nonnegative function on F(R) × F(R) gives as follows:

$$D_{p,q}(\widetilde{A},\widetilde{B})=\begin{cases}\left[(1-q)\int_0^1\left|A_\alpha^- - B_\alpha^-\right|^p d\alpha+q\int_0^1\left|A_\alpha^+ - B_\alpha^+\right|^p d\alpha\right]^{\frac{1}{p}} & if\ p<\infty\\ (1-q)\sup\limits_{0<\alpha\le 1}\left(\left|A_\alpha^- - B_\alpha^-\right|\right)+q\inf\limits_{0<\alpha\le 1}\left(\left|A_\alpha^+ - B_\alpha^+\right|\right) & if\ p=\infty.\end{cases}$$

The analytical properties of Dp,q depend on the first parameter p, while the second parameter q of Dp,q characterizes the subjective weight attributed to the sides of the fuzzy numbers. If there are no reason to distinguish any side of fuzzy numbers, $D_{p,\frac{1}{2}}$ is recommended.(for more information see [11])

***Definition 2.3.***

A mapping $\widetilde{X}:\Omega\to F(\mathrm{R})$ is said to be a fuzzy random variable associated with $(\Omega, A)$ if and only if $(\omega, x): x \in X_\alpha(\omega) \in A \times B$, where B denote the $\sigma$-field of Borel set in R.

*Definition 2.4.* The central $D_{2,q}$-mean square dispersion of $\widetilde{X}$ about $\widetilde{E}(\widetilde{X})$ (or $\widetilde{\mu}_{\widetilde{X}}$) is called $DVAR(\widetilde{X})$ given by the value (if it exists)

$$D\operatorname{var}(\widetilde{X}) = E([D_{2,q}(\widetilde{X}, \widetilde{\mu}_{\widetilde{X}})]^2) = \int_\Omega \left[ (1-q)\int_0^1 \left( X_\alpha^-(w) - (\mu_{\widetilde{X}})_\alpha^- \right)^2 d\alpha + q\int_0^1 \left( X_\alpha^+(w) - (\mu_{\widetilde{X}})_\alpha^+ \right)^2 d\alpha \right] dp(w)$$

Assume that $\widetilde{A}$ and $\widetilde{B}$ are triangular fuzzy numbers: $\widetilde{A} = T(a_1, a_2, a_3)$ and $\widetilde{B} = T(b_1, b_2, b_3)$, the $\alpha$-cuts of $\widetilde{A}$ and $\widetilde{B}$ are as follows,

$$A_\alpha = [(1-\alpha)a_1 + a_2\alpha, a_3\alpha + (1-\alpha)a_4],$$
$$B_\alpha = [(1-\alpha)b_1 + b_2\alpha, b_3\alpha + (1-\alpha)b_4].$$

It can establish that

$$\left[ D_{2,\frac{1}{2}}(\widetilde{A}, \widetilde{B}) \right]^2 = \frac{1}{6}\left[ (b_1 - a_1)^2 + 2(b_2 - a_2)^2 + (b_3 - a_3)^2 + (b_1 - a_1)(b_2 - a_2) + (b_3 - a_3)(b_2 - a_2) \right]$$

*Proposition 1:*

Suppose that $\widetilde{X}$ be a fuzzy random variable and $\widetilde{T} \in F(R)$

$$E\left[D_{2,q}\left(\widetilde{X}, \widetilde{T}\right)\right]^2 = E\left[D_{2,q}\left(\widetilde{X}, \widetilde{\mu}_{\widetilde{X}}\right)\right]^2 + \left[D_{2,q}\left(\widetilde{\mu}_{\widetilde{X}}, \widetilde{T}\right)\right]^2 = D\operatorname{var}\left(\widetilde{X}\right) + \left[D_{2,q}\left(\widetilde{\mu}_{\widetilde{X}}, \widetilde{T}\right)\right]^2$$

(2.3)

Proof:

$$E\left[D_{2,q}\left(\widetilde{X}, \widetilde{T}\right)\right]^2 = E\left[ (1-q)\int_0^1 (X_\alpha^-(w) - T_\alpha^-)^2 \, d\alpha + q\int_0^1 (X_\alpha^+(w) - T_\alpha^+)^2 \, d\alpha \right]$$

$$E\left[D_{2,q}\left(\widetilde{X}, \widetilde{T}\right)\right]^2 = (1-q)\int_0^1 E\left[(X_\alpha^-(w) - T_\alpha^-)\right]^2 d\alpha + q\int_0^1 E\left[(X_\alpha^+(w) - T_\alpha^+)\right]^2 d\alpha$$

$$= (1-q)\int_0^1 \left[\operatorname{var}(X_\alpha^-) + ((\mu_{\widetilde{X}})_\alpha^- - T_\alpha^-)\right]^2 d\alpha + q\int_0^1 \left[\operatorname{var}(X_\alpha^-) + ((\mu_{\widetilde{X}})_\alpha^- - T_\alpha^-)\right]^2 d\alpha = D\operatorname{var}(\widetilde{X}) + \left[D_{2,q}(\widetilde{\mu}_{\widetilde{X}}, \widetilde{T})\right]^2$$

Similarly, it can establish that

$$\frac{1}{n}\sum_{i=1}^{n}\left[D_{2,q}\left(\widetilde{X}_i, \widetilde{T}\right)\right]^2 = \frac{1}{n}\sum_{i=1}^{n}\left[D_{2,q}\left(\widetilde{X}, \widetilde{\overline{X}}\right)\right]^2 + \left[D_{2,q}\left(\widetilde{\overline{X}}, \widetilde{T}\right)\right]^2 = \hat{D}\operatorname{var}\left(\widetilde{X}\right) + \left[D_{2,q}\left(\widetilde{\overline{X}}, \widetilde{T}\right)\right]^2, \quad (2.4)$$

where $\hat{D}\operatorname{var}\left(\widetilde{X}\right)$ is estimator of $D\operatorname{var}\left(\widetilde{X}\right)$, such as

$$\hat{D}\operatorname{var}\left(\widetilde{X}\right) = \frac{1}{n}\sum_{i=1}^{n}\left[D_{2,q}\left(\widetilde{X}, \widetilde{\overline{X}}\right)\right]^2.$$

## 3. TRADITIONAL PROCESS CAPABILITY INDICES

One of the proposed definitions on process capability index consider that as the ratio of the real performance of process to requested performance, that is,

$$C_p = \frac{USL - LSL}{6\sigma}.$$

In order to reflect departures from the target value $(M = \frac{USL + LSL}{2})$ as well as changes in the process variation several order indices have been proposed such as $C_{pk}$ and $C_{pm}$ given as

$$C_{pk} = C_p(1-k), k = \frac{|\mu - M|}{\frac{USL - LSL}{2}},$$

and

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - M)^2}},$$

where $\mu$ is the distribution center of characteristic $X$. M is not always equal to target. $C_{pk}$ measures the distance between the process mean and the closest specification limit relation to the one-side actual process spread $3\sigma$. Departures from the target value carry more weight with the other well-known capability index $C_{pm}$. In principal, $C_{pm}$ behaved like $C_{pk}$ but $C_{pm}$ is bounded above as $\sigma \to 0$ and $\mu \neq M$. For $\mu = M$ it holds $C_p = C_{pk} = C_{pm}$. (see [11])

## 4. FUZZY PROCCESS CAPABILITY INDICES

If we define the specification limits by fuzzy quantities, it is more appropriate to define the process capability indices as fuzzy numbers.

*Definition 4.1.*
Let $L = T(l_1, l_2, l_3)$, $U = T(u_1, u_2, u_3)$ are lower and upper specification limits, where $u_1 \geq l_3$. $T = T(t_1, t_2, t_3)$ is a fuzzy target and observations are triangular fuzzy number also. Then $\tilde{C}pm$ is defined as follow (see[5])

$$\tilde{C}_{pm} = k \otimes T(u_1 - l_3, u_2 - l_2, u_3 - l_1), \quad (4.1)$$

where

$$k = \left(6\sqrt{E\left(D_{2,\frac{1}{2}}(\tilde{X}, \tilde{T})\right)^2}\right)^{-1}.$$

The point estimate for $\tilde{C}pm$ is as follow,

$$\hat{\tilde{C}}_{pm} = \hat{k} \otimes T(u_1 - l_3, u_2 - l_2, u_3 - l_1), \quad (4.2)$$

Where

$$\hat{k} = \left(6\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(D_{2,\frac{1}{2}}(\tilde{X}_i, \tilde{T})\right)^2}\right)^{-1}.$$

## 5. RANKING FUNCTION

In this paper we are going to give a fuzzy confidence interval, where comparing fuzzy numbers is emergent and so an ordering approach is needed. We need a criterion for comparison of two fuzzy subsets. A simple but efficient approach for the ordering of the elements of $F(R)$ is to define a ranking function $R: F(R) \to R$ which maps each fuzzy number into the real line, where a natural order exists, see [4]. Define the order $\underset{R}{\leq}$ on $F(R)$ by

$\tilde{A} \underset{R}{\geq} \tilde{B}$ if and only if $R(\tilde{A}) \geq R(\tilde{B})$

$\tilde{A} \underset{R}{\leq} \tilde{B}$ if and only if $R(\tilde{A}) \leq R(\tilde{B})$

$\tilde{A} \underset{R}{=} \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$

Where $\tilde{A}$ and $\tilde{B}$ are in $F(R)$.

Several ranking functions have been proposed by researchers to suit their requirements of the problems under consideration. For more details see [1,10]. The ranking function proposed by Roubens [3,8] is defined by

$$R_r(\widetilde{A})=\frac{1}{2}\int_0^1\left(\inf \widetilde{A}_\alpha+\sup \widetilde{A}_\alpha\right)d\alpha .$$
(5.1)

From now on, if $R_r$ is the Roubens's ranking function, then we write $\underset{R}{\leq}$ simply as $\leq$. We can easily prove the following lemmas (for proof see[5]).

*Lemma 1.*
If $T(a,b,c)\in F_T(R)$ then Roubens's ranking function reduces to following

$$R_r(T(a,b,c))=\frac{2b+a+c}{4} .$$
(5.2)

*Lemma 2.*
Let $m,n\in R$ , $T(a,b,c)\in F_T(R)$ and $2b+a+c\geq 0$. Then according to Roubens's ranking Function $m\otimes T(a,b,c)\leq n\otimes T(a,b,c)$ if and only if $m\leq n$ .

## 6. FUZZY CONFIDENCE INTERVAL FOR $\widetilde{C}_{pm}$

*Definition 6.1.*
Let $A,B\in F_T(R)$ and $A\leq B$ . The fuzzy interval $[A,B]$ is the set
$[A,B]=\{C\in F_T(R)\mid A\leq C\leq B\}$
Note that $[A,B]$ is nonempty, since $A,B\in[A,B]$. Suppose that the set of all random samples of size $n$ which are possible is $X^{(n)}$ .

*Definition 6.2.*
Any function $A:X^{(n)}\to F_T(R)$ is called a fuzzy statistic. Note that $A(X_1,...,X_n)$ only depends on $X_1,...,X_n$ and without any unknown parameters. When the observation $X=(X_1,...,X_n)$ is given, then the value of the statistic, $A(X)$ is just a triangular fuzzy number. Let $X$ be a measurable random variable on the probability space $(\Omega,F,P)$ and $T=T(a,b,c)\in F_T(R)$ such that $2b+a+c\geq 0$. We define
$(X\otimes T)(\omega)=X(\omega)\otimes T,\forall\omega\in\Omega$
according to Eq. (2).

*Proposition 2.*
Let $X$ be a measurable random variable on the probability space $(\Omega,F,P)$, $k_1,k_2\in R$ and $T=T(a,b,c)\in F_T(R)$ with $2b+a+c\geq 0$. Then
$pr(k_1\otimes T\leq X\otimes T\leq k_2\otimes T)=1-\alpha$

If and only if
$pr(k_1\leq X\leq k_2)=1-\alpha$ .

*Definition 6.3.*
Let $A$ and $B$ be fuzzy statistics as triangular fuzzy numbers, where $A\leq B$. Then $[A,B]$ is a $100(1-\alpha)\%$ fuzzy confidence interval for $\widetilde{\theta}=X\otimes T$, where $P(A\leq\widetilde{\theta}\leq B)=1-\alpha$ .

*Theorem 1.*
Suppose that $\widetilde{X}_1,\widetilde{X}_2,...,\widetilde{X}_n$ are independent , identically distributed fuzzy random variables with $N(\mu,\sigma^2)$ and $L=T(l_1,l_2,l_3)\in F_T(R)$ , $U=T(u_1,u_2,u_3)\in F_T(R)$ , are lower and upper specification limits (engineering fuzzy limits) , where $u_1\geq l_3$ . taget value is triangular fuzzy number also such as $T=T(t_1,t_2,t_3)\in F_T(R)$. Then the following interval is a $100(1-\alpha)\%$ fuzzy

confidence interval for $\widetilde{C}_{pm}$

$$\left(\hat{\widetilde{C}}_{pm} \otimes \sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)}{n(1+\hat{\delta})}}, \hat{\widetilde{C}}_{pm} \otimes \sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)}{n(1+\hat{\delta})}}\right)$$

Where $\hat{\widetilde{C}}_{pm}$ is defined with Eq.(4.2) ,

$\delta = \dfrac{[D_{2,1/2}(\widetilde{\mu},\widetilde{T})]^2}{D\text{var}}$ and

$\chi'^2_{n,\alpha/2}(n\delta)$ depicted as the non-central chi-square with n degrees of freedom and Non-centrality parameter $n\delta$ .

*Proof:*

The statistic $\sum_{i=1}^{n} \dfrac{[D_{2,1/2}(\widetilde{X}_i,\widetilde{T})]^2}{D\text{ var}}$ is distributed as the non- central chi-square with n degrees of freedom and Non-centrality parameter $n\delta$ where $\delta = \dfrac{[D_{2,1/2}(\widetilde{\mu},\widetilde{T})]^2}{D\text{var}}$ [9]. We have

$$P\left(\chi'^2_{n,\alpha/2}(n\delta) < \frac{\sum_{i=1}^{n}[D_{2,\frac{1}{2}}(\widetilde{X}_i,\widetilde{T})]^2}{D\text{ var}(\widetilde{X})} < \chi'^2_{n,1-\alpha/2}(n\delta)\right) = 1-\alpha$$

Therefore

$$P\left(\frac{\chi'^2_{n,\alpha/2}(n\delta)D\text{ var}(\widetilde{X})}{\sum\left[D_{2,\frac{1}{2}}\left(\widetilde{X}_i,\widetilde{T}\right)\right]^2 E\left[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})\right]^2} < \frac{1}{E\left[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})\right]^2} < \frac{\chi'^2_{n,1-\alpha/2}(n\delta)D\text{ var}(\widetilde{X})}{\sum\left[D_{2,\frac{1}{2}}\left(\widetilde{X}_i,\widetilde{T}\right)\right]^2 E\left[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})\right]^2}\right) = 1-\alpha .$$

By proposition 1(Eq.(2.3)), we can write

$$P\left(\sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)\hat{D}\text{ var}(\widetilde{X})}{n\hat{D}\text{ var}(\widetilde{X}) + n[D_{2,\frac{1}{2}}(\overline{\widetilde{X}},\widetilde{T})]^2}} \times \frac{1}{6\sqrt{1/n\sum[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})]^2}} < \frac{1}{6\sqrt{E[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})]^2}} < \sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)\hat{D}\text{ var}(\widetilde{X})}{n\hat{D}\text{ var}(\widetilde{X}) + n[D_{2,\frac{1}{2}}(\overline{\widetilde{X}},\widetilde{T})]^2}} \times \frac{1}{6\sqrt{1/n\sum[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})]^2}}\right) = 1-\alpha .$$

Let

$$k_1 = \sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)\hat{D}\text{ var}(\widetilde{X})}{n\hat{D}\text{ var}(\widetilde{X}) + n[D_{2,\frac{1}{2}}(\overline{\widetilde{X}},\widetilde{T})]^2}} \times \frac{1}{6\sqrt{1/n\sum[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})]^2}}$$

and

$$k_2 = \sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)\hat{D}\text{ var}(\widetilde{X})}{n\hat{D}\text{ var}(\widetilde{X}) + n[D_{2,\frac{1}{2}}(\overline{\widetilde{X}},\widetilde{T})]^2}} \times \frac{1}{6\sqrt{1/n\sum[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})]^2}}$$

.

By proposition 2 and the fact $u_1 \geq l_3$ ,we can obtain

$$p(k_1 \otimes T(u_1-l_3,u_2-l_2,u_3-l_1) < \frac{1}{6\sqrt{E[D_{2,\frac{1}{2}}(\widetilde{X},\widetilde{T})]^2}} \otimes T(u_1-l_3,u_2-l_2,u_3-l_1) < k_2 \otimes T(u_1-l_3,u_2-l_2,u_3-l_1)) = 1-\alpha$$

By Eq. (2.1)

$$P\left(T\left(k_1(u_1-l_3),k_1(u_2-l_2),k_1(u_3-l_1)\right) < \frac{1}{6\sqrt{E\left[D_{2,\frac{1}{2}}\left(\widetilde{X},\widetilde{T}\right)\right]^2}} \otimes T\left(u_1-l_3,u_2-l_2,u_3-l_1\right) < T\left(k_2(u_1-l_3),k_2(u_2-l_2),k_2(u_3-l_1)\right)\right) = 1-\alpha$$

By Eq.(4.1) and Eq.(4.2) we obtian

$$P\left(\sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)}{n(1+\hat{\delta})}} \otimes \hat{\widetilde{C}}_{pm} < \widetilde{C}_{pm} < \sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)}{n(1+\hat{\delta})}} \otimes \hat{\widetilde{C}}_{pm}\right) = 1-\alpha$$

Hence by definition (6.3)

$$\left(\sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)}{n(1+\hat{\delta})}} \otimes \hat{\widetilde{C}}_{pm}, \sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)}{n(1+\hat{\delta})}} \otimes \hat{\widetilde{C}}_{pm}\right)$$

is a $100(1-\alpha)\%$ fuzzy confidence interval for $\widetilde{C}_{pm}$ .

Note that any $\widetilde{C}_{pm} = T(a,b,c)$ ,with

$$R_r\left(\widetilde{C}_{pm}\right) \in \left(R_r\left(\sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)}{n(1+\hat{\delta})}} \otimes \hat{\widetilde{C}}_{pm}\right), R_r\left(\sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)}{n(1+\hat{\delta})}} \otimes \hat{\widetilde{C}}_{pm}\right)\right)$$

is in the $100(1-\alpha)\%$ fuzzy confidence interval given in theorem1 if the following inequalities hold

$$\frac{u_1 - l_3}{6\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[D_{2,\frac{1}{2}}\left(\tilde{X}_i,\tilde{T}\right)\right]}}\sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)}{n(1+\hat{\delta})}} \le a,$$

$$\frac{u_2 - l_2}{6\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[D_{2,\frac{1}{2}}\left(\tilde{X}_i,\tilde{T}\right)\right]}}\sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)}{n(1+\hat{\delta})}} < b < \frac{u_2 - l_2}{6\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[D_{2,\frac{1}{2}}\left(\tilde{X}_i,\tilde{T}\right)\right]}}\sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)}{n(1+\hat{\delta})}}$$

$$\sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)}{n(1+\hat{\delta})}} < \frac{6b\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[D_{2,\frac{1}{2}}\left(\tilde{X}_i,\tilde{T}\right)\right]}}{u_2 - l_2} < \sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)}{n(1+\hat{\delta})}}$$

ii) Let $\lambda \in \left(\sqrt{\frac{\chi'^2_{n,\alpha/2}(n\delta)}{n(1+\hat{\delta})}}, \sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)}{n(1+\hat{\delta})}}\right)$

Then any

$\tilde{C}_{pm} = \lambda \otimes \hat{\tilde{C}}_{pm}$ is in the $100(1-\alpha)\%$ fuzzy confidence interval given in Theorem1.

$$c \le \frac{u_3 - l_1}{6\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left[D_{2,\frac{1}{2}}\left(\tilde{X}_i,\tilde{T}\right)\right]}}\sqrt{\frac{\chi'^2_{n,1-\alpha/2}(n\delta)}{n(1+\hat{\delta})}},$$

and

## 7. A NUMERICAL EXAMPLE

Table 1 shows the Fuzzy data given in [1]. In this example, $LSL = T\ (5.4, 5.5, 5.6)$, $USL = T\,(6.3,\ 6.4, 6.5)$ and $T = T\,(5.9, 6, 6.1)$. Fig 1(a) shows the membership function of fuzzy process specification limits. The value of $\tilde{C}_{pm}$ based on the definition 2.5 is $T = T\,(0.5823, 0.7487, 0.9151)$ as shown in Fig1(b). We present $100(1-\alpha)\%$ confidence intervals using the Theorem1 in Table II and illustrate these fuzzy confidence intervals in Fig1(c).

TABLE I. 30 triangular fuzzy observations

| | | |
|---|---|---|
| $\tilde{X}_1$ =T(5.85,6.15,6.35) | $\tilde{X}_{11}$ =T(5.86,6.04,6.25) | $\tilde{X}_{21}$ =T(5.5,5.81,5.99) |
| $\tilde{X}_2$ =T(5.79,5.9,5.98) | $\tilde{X}_{12}$ =T(6.13,6.23,6.33) | $\tilde{X}_{22}$ =T(5.6,5.92,6.05) |
| $\tilde{X}_3$ =T (5.71,5.83,5.99) | $\tilde{X}_{13}$ =T(5.95,6.05,6.19) | $\tilde{X}_{23}$ =T(5.5,5.75,5.95) |
| $\tilde{X}_4$ =T (6.05,6.18,6.32) | $\tilde{X}_{14}$ =T(5.06,5.65,5.70) | $\tilde{X}_{24}$ =T(5.84,6.03,6.15] |
| $\tilde{X}_5$ = T(5.89,6.06,6.23) | $\tilde{X}_{15}$ =T(5.65,5.74,5.84) | $\tilde{X}_{25}$ =T(6.05,6.30,6.50] |
| $\tilde{X}_6$ = T(6.01,6.10,6.25) | $\tilde{X}_{16}$ =T(5.70,5.77,5.83) | $\tilde{X}_{26}$ =T(6.25,6.35,6.45] |
| $\tilde{X}_7$ =T(6.15,6.20,6.30) | $\tilde{X}_{17}$ =T(6.23,6.32,6.40) | $\tilde{X}_{27}$ =T(5.65,5.86,6.05] |
| $\tilde{X}_8$ = T(5.64,5.81,6.05) | $\tilde{X}_{18}$ =T(5.60,5.70,5.08) | $\tilde{X}_{28}$ =T(5.70,5.87,5.95] |
| $\tilde{X}_9$ =T(5.8,5.9,5.98) | $\tilde{X}_{19}$ =T(95.85,5.95,6.05) | $\tilde{X}_{29}$ =T(5.75,5.95,5.15] |
| $\tilde{X}_{10}$ =T(6.01,6.12,6.24) | $\tilde{X}_{20}$ =T(5.90,6.00,6.10) | $\tilde{X}_{30}$ =T(6.10,6.23,6.46] |

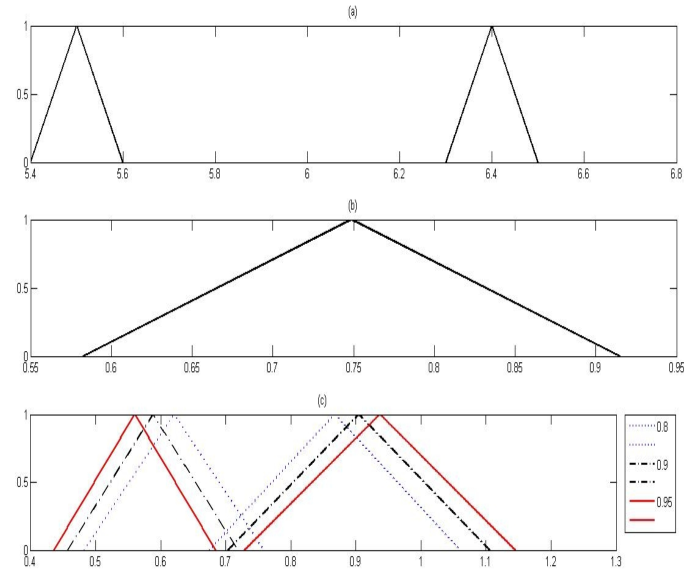


***Figure 1: Graphs of membership functions***

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