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AN INTERVAL LINEAR PROGRAMMING APPROACH FOR UNCERTAINTY-BASED DECISION MAKING IN VEHICLE RECYCLING INDUSTRY

Abstract: *In a vehicle recycling system, the all modeling data can hardly be expressed or obtained in deterministic form. However, they easily can be obtained as interval values. When the parameters are known only within certain bounds (i.e., intervals), the approach to tackling such problems is called interval linear programming. There is a lack of research of uncertainties that exist in vehicle recycling industry, and no previous research was reported on interval-based programming for vehicle recycling planning. Having that in mind, the objective of this paper is to develop an interval linear programming based model for optimal production planning in a vehicle recycling industry. The proposed model provides optimal solutions to fully interval vehicle recycling planning problem which helps the decision makers to analyze economic activities and to arrive at the best managerial decisions.*

Keywords: *Interval Linear Programming, End-of-Life Vehicles, Recycling, Waste Management*

1. INTRODUCTION

It has been recognized that deterministic optimization techniques, such as linear programming (LP), are not sufficient to model complex environmental engineering problems, particular its uncertain features. In order to solve a LP model, its parameters must be fixed at specific values, which imply that they are perfectly accurate. In practice, however, the data used in LP models is often clouded with uncertainty. If we want to understand the impact of the uncertainty in data, we have to upgrade present LP models and incorporate uncertainty.

When the parameters are known only within certain bounds, the approach to tackling such problems is called interval linear programming (ILP). It represents an extension of the classical LP problem to an inexact environment. ILP method can deal with the uncertain parameters expressed as intervals without any distributional information that is always required in for instance, fuzzy and stochastic programming. ILP allows the interval information to be directly communicated into the optimization process and resulting solution.

In a vehicle recycling system, the all modeling data can hardly be expressed or obtained in deterministic form. However, they easily can be obtained as interval values.

There is a lack of research of uncertainties that exist in vehicle recycling industry [1-9]. In addition, no previous research was reported on interval-based programming for vehicle recycling planning [4-5, 7-8]. Therefore, the objective of this paper is to develop an interval linear programming based model for optimal production planning in a vehicle recycling industry.

This paper is organized in the following way: in Section 2 the basic methodology for dealing with uncertainities in environmental engineering is shortly reviewed; in Section 3 developed model for optimal production planning in a vehicle recycling industry is presented; and concluding remarks are outlined in Section 4.

2. METHODOLOGY

To better reflect uncertainties in environmental engineering, several optimization techniques were developed. They include:

- *Fuzzy mathematical programming* (FMP). FMP can be categorized into two major streams:
  - *Fuzzy flexibility programming* (FFP). In FFP, the flexibility in the constraints and fuzziness in the objective are represented by fuzzy sets and denoted as "fuzzy constraints" and "fuzzy goal" respectively, which can be expressed as membership grades. However, FFP could hardly tackle uncertainties expressed as ambiguous coefficients in the objective function and constraints.
  - *Fuzzy possibility programming* (FPP). In FPP, fuzzy parameters are introduced into the modelling framework, and these parameters are presented as fuzzy sets with possibility distributions. The limitation of FPP is that when many uncertain parameters are expressed as fuzzy sets in a model, interactions among these uncertainties may lead to serious complexities, particularly for large-scale practical problems.
- *Stochastic mathematical programmmeng* (SMP). SMP is derived from probability theory. In SMP, random elements are introduced to account for probabilistic uncertainty in the coefficients. The major strength of SMP method is that it allows decision makers to have a complete view of the effects of uncertainties and the relationships between uncertain inputs and resulting solutions. The major problem of SMP is that no sufficient data are available to obtain the probability distribution functions (PDFs) for random parameters. In addition, even if these functions are available, it is extremely hard to solve a large scale stochastic management system planning problem with all uncertain data being expressed as PDFs.
- *Interval mathematical programming* (IMP). When the parameters are known only within certain bounds, the approach to tackling such problems is called IMP or robust optimization. Compared to fuzzy and stochastic programming, in terms of data quality and requirements, IMP does not need the information of membership functions or distribution of parameters which may be hard to obtain in practical applications. Moreover, fuzzy and stochastic methods often lead to more complicated sub-models and may not be practical for many real life situations.

Interval linear programmmming (ILP) is one of the IMP which can effectively deal with uncertainties without leading to more complicated sub-models. Because of this characteristic, this optimization technique is used to develop a long-term model for optimal production planning in a vehicle recycling industry.

2.1 Interval linear programming

A typical ILP model can be defined as follows:

$$\text{Maximize } f^{\pm} = C^{\pm}X^{\pm} \quad (1)$$

Subject to:

$$A^{\pm}X^{\pm} - B^{\pm} (\geq)(=,\leq) 0 \quad (2)$$

$$X^{\pm} \geq 0 \quad (3)$$

where ± = the interval number with known upper-bound (superscript +) and lower-bound (superscript −), but unknown distribution information; $f^{\pm}$ = objective function; $A^{\pm} = \{a_{ij}^{\pm}\}$ and $C^{\pm} = [c_1^{\pm}, c_2^{\pm}, ..., c_n^{\pm}]$ are coefficients (i=1, …,m, j=1,2,…,n); $X^{\pm} = [x_1^{\pm}, x_2^{\pm}, ..., x_n^{\pm}]^T$ = the unknown decision variables; and $B^{\pm} = [b_1^{\pm}, b_2^{\pm}, ..., b_m^{\pm}]^T$ = the right-hand constraints.

Eqs. (1)-(3) can be decomposed into two submodels corresponding to the lower and upper bounds of the objective function and solved using one of the standard ILP algorithms. Among various different approaches for solving the ILP problem, gray LP (GLP) algorithm and best worst case (BWC) analysis represent two major algorithms that are computationally efficient in obtaining interval solutions. Both algorithms reformulate the original model using extreme constraints to represent the most conservative and the most aggressive conditions.

The BWC algorithm is also a two-step method which obtains the optimal solution through solving two submodels corresponding to the lower and upper bound of objective function. The first step of the BWC algorithm is to formulate the submodel corresponding to the upper bound of the objective function as

$$\text{Maximize } f^{+} = \sum_{j=1}^{n} c_j^{+} x_j \quad (1a)$$

Subject to:

$$\sum_{j=1}^{n} a_{ij}^{-} x_j - b_i^{+} \leq 0,\ i=1,2,...,m \quad (2a)$$

$$x_j \geq 0,\ j=1,2,...,n \quad (3a)$$

and the second step involves formulating and solving the following model:

$$\text{Maximize } f^{-} = \sum_{j=1}^{n} c_j^{-} x_j \quad (1b)$$

Subject to:

$$\sum_{j=1}^{n} a_{ij}^{+} x_j - b_i^{-} \leq 0,\ i=1,2,...,m \quad (2b)$$

$$x_j \geq 0,\ j=1,2,...,n \quad (3b)$$

The major difference between the two algorithms lies in that the GLP algorithm differentiates the selection of extreme parameter values (i.e., lower or upper bounds of coefficients) for decision variables in the objective function based on their different signs (i.e., negative or positive coefficients in c_j), while the BWC treats all the parameters without discrimination. Both algorithms provide an interval solution space, and each point in the interval solution space becomes a potential solution to form a decision alternative for implementation. Due to solution space decrease, the GLP algorithm arbitrarily ignores some system uncertainties to a certain degree and this ignorance was not theoretically or mathematically justified. In this sense, the BWC algorithm seems to be a better method in handling the uncertainties presented as intervals. Having that in mind, it can be concluded that GLP algorithm ignores some of the system uncertainties when reformulating the sub-model constraints and this treatment could be a potential flaw of this algorithm and could very possibly lead to feasibility and optimality concerns towards the generated interval optimal solutions.

3. AN INTERVAL LINEAR PROGRAMMING BASED MODEL FOR OPTIMAL PRODUCTION PLANNING IN A VEHICLE RECYCLING INDUSTRY

The proposed model tackles a long-term vehicle recycling planning problem. Its objective is to maximise the vehicle recycling factory's profit over the planning horizon

(4) Objective function:
Maximize *Profit* = Total revenue (*TR*) - Total cost (*TC*)

(4.1) Total revenue:

$$TR = \sum_{t=1}^{T} \sum_{i=I-|M|-1}^{I-1} \sum_{i' \in \Omega_i} R^{\pm}_{i'it} X_{i'it}$$

(4.2) Total cost:
TC = Hulk procurement cost (*CP*)
+ Storage cost (*CI*)
+ Sorting cost (*CS*)
+ Transportation cost (*CT*)
+ Thermal treatment cost (*CA*)
+ Landfill disposal cost (*CL*)

(4.2.1) Hulk procurement cost:

$$CP = \sum_{t=1}^{T} CP^{\pm}_t P_t$$

(4.2.2) Storage cost:

$$CI = \sum_{t=1}^{T} Z^{\pm}_t CP^{\pm}_t S_t$$

(4.2.3) Sorting cost:

$$CS = \sum_{t=1}^{T} \sum_{i=1}^{I-I'-2} CS^{\pm}_{it} \sum_{i' \in \Omega_i} X_{i'it}$$

(4.2.4) Transportation cost:

$$CT = \sum_{t=1}^{T} \sum_{i=I-I'}^{I-1} \sum_{i' \in \Omega_i} CT^{\pm}_{i'i} X_{i'it}$$

(4.2.5) (Advanced) thermal treatment cost:

$$CA = \sum_{t=1}^{T} \sum_{i \in D} CA^{\pm}_{it} \sum_{i' \in \Omega_i} X_{i'it}$$

(4.2.6) Landfill disposal cost:

$$CL = \sum_{t=1}^{T} CL^{\pm}_t \sum_{i \in \Omega_{12}} X_{i12t}$$

Constraints:

(5) Inventory balances equations:

$$S_t = \begin{cases} P_t - X_{01t}, & \text{if } t=1 \\ P_t + S_{t-1} - X_{01t}, & \text{if } t=2,...,T \end{cases}$$

(6) Safety stock level:

$$S_t \geq S^{\pm}_{min}, \quad t = 1,...,T$$

(7) Sorting capacity:

$$\sum_{i' \in \Omega_i} X_{i'it} \leq C^{\pm}_{it},$$
$$i = 1,...,I - I' - 2; \; t = 1,...,T$$

(8) Material flow balances equations:

$$\sum_{j' \in \Psi_j} X_{ij't} = E_{ijt} \sum_{i' \in \Omega_i} X_{i'it},$$
$$i = 1,...,I - I' - 2; \; j = \mathrm{A}_i; \; t = 1,...,T$$

(9) Mixing operations:

$$X_{ijt} = \sum_{i' \in \Omega_i} X_{i'it},$$
$$i = I - I' - 1; \; j \in \Phi_i; \; t = 1,...,T$$

(10) Recycling rate:

$$\sum_{i \in M} \sum_{i' \in \Omega_i} X_{i'it} + \sum_{i \in F} ER^{\pm}_i \sum_{i' \in \Omega_i} X_{i'it} \geq Q_R X_{01t},$$
$$t = 1,...,T$$

(11) Recovery rate:

$$\sum_{i \in M} \sum_{i' \in \Omega_i} X_{i'it} + \sum_{i \in F} ER^{\pm}_i \sum_{i' \in \Omega_i} X_{i'it} + \sum_{i \in D} EE^{\pm}_i \sum_{i' \in \Omega_i} X_{i'it} \geq X_{01t} Q_{R'}, \quad t = 1,...,T$$

(12) Energy recovery rate:

$$\sum_{i \in D} EE_i^{\pm} \sum_{i' \in \Omega_i} X_{i'it} \leq Q_E X_{01t}, \quad t = 1,...,T$$

(13) Value domains of decision variables:

$$P_t \geq 0, S_t \geq 0, \quad t = 1,...,T$$

$$X_{ijt} \geq 0,$$
$$i = 0,...,I - I' - 1; j \in \Phi_i; t = 1,...,T$$

where i = index of entity; j = index of material flow; t = index of time period; A_i = set of material flows isolated with sorting entity i; Ψ_j = set of entities on which material flow j is forwarded; Ω_i = set of entities that route materials to entity i; Φ_i = set of entities on which materials are routed from entity i; F = set of destinations where material recycling take place; M = set of various metal producers in EU member state; D = set of destinations where energy recovery takes place; I = number of entities; J = number of material flows; T = number of analysed time periods; I' = number of destinations; $S_{min}^{\pm}$ = safety inventory level, (tonne); $C_{it}^{\pm}$ = processing capacity in the case of entity i and period t, (tonne/h); $ER_i^{\pm}$ = recycling efficiency of destination i, (%); $EE_i^{\pm}$ = energy efficiency of destination i in, (%); E_{ijt} = efficiency of sorting entity i in the case of material flow j and period t, (%); Q_R = EU recycling quota, (%); $Q_{R'}$ = EU recovery quota, (%); Q_E = EU energy quota, (%); $R_{i'it}^{\pm}$ = revenue from metal fraction sorted on entity i and sold to destination i' in time period t, (€/tonne); $CA_{it}^{\pm}$ = thermal treatment cost in destination plant i and period t, (€/tonne); $CL_t^{\pm}$ = landfill disposal cost in period t, (€/tonne); $CP_t^{\pm}$ = procurement cost in period t, (€/tonne); $Z_t^{\pm}$ = percentage of capital cost for inventory in period t; $CS_{it}^{\pm}$ = sorting cost in the case of entity i and period t, (€/tonne); $CT_{i'it}^{\pm}$ = transportation cost from entity i' to i in period t, (€/tonne); S_t = weight of vehicle hulks in storage at the end of period t, (tonne); P_t = weight of incoming procurement in period t, (tonne); and $X_{ii't}$ = weight of material flow routed from entity i to i' in period t, (tonne).

4. CONCLUSION

In this paper we developed an interval linear programming based model for optimal production planning in a vehicle recycling industry. The model provides optimal solutions to fully interval vehicle recycling planning problem which helps the decision makers to analyze economic activities and to arrive at the best managerial decisions.

The proposed model was an attempt for long-term planning vehicle recycling systems through incorporating interval optimization approach within a general linear programming framework. It can be of assistance not only to European vehicle recyclers with the aim of creating long-term production plans and improving their ecological and economical efficiency, but also to EU policy makers in order to analyse the ELV Directive [10] influence on vehicle industry behaviour.

Future research directions will focus on extensive testing of the proposed model and creating strategic production guidelines for EU vehicle recycling factories, and making capacity strategies for industrial lanfill sites, municipal solid waste incinerators and advanced thermal treatment plants.

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