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INFLUENCE OF MESH QUALITY ON FLUID FLOW CALCULATED WITH SOFTWARE PAK-F EXPLICIT

Abstract: *Paper presents an overview on study of blood flow in the human body. Basic equations of fluid flow are presented and the basis for creating software package PAK-F Explicit for the calculation of viscous fluid flow. Geometry of the carotid artery is obtained through the analysis of images from CT scanner. Finite element models are created on real 3d geometry model. Presented results show the influence of finite element mesh quality on hemodynamic parameters of blood flow.*

Keywords: *Blood flow, FEM, Explicit, Bifurcation, Wall shear stress*

1. INTRODUCTION

Biomechanics is a relatively new scientific discipline that has emerged from several scientific disciplines such as medicine, engineering, computational science. Expansion of the computer sciences brings biomechanics to a new level higher than the level of about 20 years ago. Software for the management of biomechanical characteristics have evolved along with computers. In recent years there are more software programs that are based on calculations of blood flow in arteries, air flow in the lungs and stress conditions in human skeletal system. Good management of these technologies may lead to research improvement but also to cutting costs of research, increasing the field of application, etc.

Atherosclerosis is one of the most widespread diseases that affecting blood vessels in the human body. The studies presented in [1-4] shows that very responsible are hemodynamic factors such as low or reversed wall shear stress. Computational fluid dynamics (CFD) is an area of fluid dynamics that can be applied to study the hemodynamic factors in human body.

The software package PAK-F [5] was developed at the Laboratory for Engineering Software, Faculty of Engineering, Kragujevac. It consists of modules for steady and transient incompressible fluid flow with heat transfer. It is based on finite element method and on the fundamental equations of viscous fluid flow. The reason for development of PAK-F Explicit in addition to PAK-F software is based on cost effectiveness and the possibility of upgrading the software in terms of solving biomechanical problems. To create an analysis file for software PAK-F Explicit, it is necessary to create a model in any pre-processors such as GID, FEMAP, CATIA etc. After fluid flow analysis results are printed in a form that can be post-processed in other software such as FEMAP, GiD, Paraview, IDEAS, PAK-G, etc.

This paper is structured as follows: basic equations of incompressible fluid flow and the development of software PAK-F Explicit are given in the section 2. In the section 3 – the case study, the blood flow through carotid artery bifurcation is presented which has intention to investigate capabilities and performances of software package PAK-F Explicit. Section 4 gives conclusions and directions for improvement of software in the sense of needs of research and software performances.

The paper has two main goals. The first one is to show that PAK-F Explicit software can obtain valuable inputs to cardiologists who are decision makers in the sense of health management. The other goal is to prove that PAK-F Explicit is qualitative tool for scientific research.

2. BASIC EQUATIONS OF INCOMPRESSIBLE VISCOUS FLUID FLOW

Basic differential equations that governing the flow of an incompressible fluid [6-8] are the Navier-Stokes equations given by expressions:

$$\rho\left(\frac{\partial \mathrm{v}_i}{\partial t}+\mathrm{v}_{i,j}\mathrm{v}_j\right)+p_{,i}-\mu \mathrm{v}_{i,jj}-\mathrm{f}_i^V=0 \qquad (1)$$

$$\frac{\partial \rho}{\partial t}+\left(\rho\,\mathrm{v}_i\right)_{,i}=0 \qquad (2)$$

Equation (1) represent the second Newton's law applied to the mass of fluid in control volume and (2) represents the continuity equation of fluid flow. In previous equations ρ is the fluid density, v is velocity of fluid, p is pressure of fluid, μ is dynamic viscosity and f_i^V is volume forces.

Using the equation of state it is considered that the pressure depends on density:

$$p=p\left(\rho\right) \qquad (3)$$

Speed of sound can be expressed in this way:

$$c^2=\frac{\partial p}{\partial \rho} \qquad (4)$$

If a material derivative is applied to pressure from equation (3) it gives the following expression:

$$\frac{Dp}{Dt}=\frac{\partial p}{\partial \rho}\left(\frac{\partial \rho}{\partial t}+\rho_{,i}\,\mathrm{v}_i\right) \qquad (5)$$

Substituting (5) into (2) and using (4) following expression is given:

$$\frac{\partial p}{\partial t}+\mathrm{v}_i\;p_{,i}+\rho c^2\,\mathrm{v}_{i,j}=0 \qquad (6)$$

Using Galerkin method, with appropriate interpolation functions:

$$\mathrm{v}_i=\mathrm{h}_I\mathrm{V}_i^I \qquad I=1,2,\ldots,N \qquad (7)$$

$$p=\hat{\mathrm{h}}_I\mathrm{P}_I \qquad I=1,2,\ldots,M \qquad (8)$$

and integration by volume of finite element, a matrix form of equations (1) and (2) is obtained such as:

$$\mathbf{M}\dot{\mathbf{v}}+\mathbf{K}_{VV}\mathbf{v}+\mathbf{K}_{VP}\mathbf{p}=\mathbf{F}_B+\mathbf{F}_S \qquad (9)$$

$$\mathbf{M}_P\dot{\mathbf{p}}+\mathbf{K}_{PV}\mathbf{v}+\mathbf{K}_{PP}\mathbf{p}=0 \qquad (10)$$

Components of this matrix and vectors are:

$$\left[\mathbf{M}_V\right]_{IJ}=\rho\int_V \mathrm{h}_I\mathrm{h}_J\,dV \qquad (11)$$

$$\left[\mathbf{K}_{VV}\right]_{IJ}=\int_V \mathrm{h}_I\mathrm{v}_j\mathrm{h}_{J,j}\,dV+ +\int_V \mu\mathrm{h}_{I,j}\mathrm{h}_{J,j}\,dV \qquad (12)$$

$$\left[\mathbf{K}_{VP}\right]_{IJ}=-\int_V \mathrm{h}_{I,i}\hat{\mathrm{h}}_J\,dV \qquad (13)$$

$$\left[\mathbf{F}_B\right]_I=\int_V \mathrm{h}_I\mathrm{f}_i^V\,dV \qquad (14)$$

$$\left[\mathbf{F}_S\right]=\int_S \mathrm{h}_I\left(-p\delta_{ij}+\mu\mathrm{v}_{i,j}\right)\mathrm{n}_j\,dS \qquad (15)$$

$$\left[\mathbf{M}_P\right]_{IJ}=\int_V \mathrm{h}_I\mathrm{h}_J\,dV \qquad (16)$$

$$\left[\mathbf{K}_{PV}\right]_{IJ}=\rho c^2\int_V \mathrm{h}_I\mathrm{h}_{J,i}\,dV \qquad (17)$$

$$\left[\mathbf{K}_{PP}\right]_{IJ}=\int_V \mathrm{h}_I\hat{\mathrm{h}}_I\;\mathrm{v}_{Ji}\;h_{J,i}\,dV \qquad (18)$$

By grouping equations (9) and (10), system of differential equations is presented as:

$$\begin{bmatrix}\mathbf{M}_V & 0\\ 0 & \mathbf{M}_P\end{bmatrix}\begin{bmatrix}\dot{\mathbf{v}}\\ \dot{\mathbf{p}}\end{bmatrix}+\begin{bmatrix}\mathbf{K}_{VV} & \mathbf{K}_{VP}\\ \mathbf{K}_{PV} & \mathbf{K}_{PP}\end{bmatrix}\begin{bmatrix}\mathbf{v}\\ \mathbf{p}\end{bmatrix}=\begin{bmatrix}\mathbf{F}_B+\mathbf{F}_S\\ 0\end{bmatrix} \qquad (19)$$

If an two-step explicit scheme for solving systems of differential equations is applied to (19) according to [9]:

1*st step*

$$\bar{\mathbf{M}}_V \mathbf{v}^{n+1/2} = \bar{\mathbf{M}}_V \mathbf{v}^n - \\ -\frac{\Delta t}{2}\left(\mathbf{K}_{VV}\mathbf{v}^n + \mathbf{K}_{VP}\mathbf{p}^n - \mathbf{F}_B{}^n - \mathbf{F}_S{}^n\right) \quad (20)$$

$$\bar{\mathbf{M}}_P \mathbf{p}^{n+1/2} = \tilde{\mathbf{M}}_P \mathbf{p}^n - \\ -\frac{\Delta t}{2}\left(\mathbf{K}_{PV}\mathbf{v}^n + \mathbf{K}_{PP}\mathbf{p}^n\right) = 0$$

2*nd step*

$$\bar{\mathbf{M}}_V \mathbf{v}^{n+1} = \bar{\mathbf{M}}_V \mathbf{v}^n - \\ -\Delta t\left(\mathbf{K}_{VV}\mathbf{v}^{n+1/2} + \mathbf{K}_{VP}\mathbf{p}^{n+1/2} - \mathbf{F}_B{}^n - \mathbf{F}_S{}^{n+1/2}\right) \quad (21)$$

$$\bar{\mathbf{M}}_P \mathbf{p}^{n+1} = \tilde{\mathbf{M}}_P \mathbf{p}^n - \\ -\Delta t\left(\mathbf{K}_{PV}\mathbf{v}^{n+1/2} + \mathbf{K}_{PP}\mathbf{p}^{n+1/2}\right) = 0$$

that gives the size of the unknown velocity v and pressure p.

In equations (20) i (21) $\bar{\mathbf{M}}_V$ i $\bar{\mathbf{M}}_P$ are diagonal mass matrix, where the matrix $\tilde{\mathbf{M}}_P$ is determined as follows:

$$\tilde{\mathbf{M}}_P = e\bar{\mathbf{M}}_P + \left(1-e\right)\mathbf{M}_P \quad (22)$$

where e is parameter which controls the numerical stability.

Wall shear stress is a hemodynamic factor which have great importance to study the problem of blood flow. In this case wall shear stress is calculated based on equation:

$$\boldsymbol{\tau}_W = -\mu \left.\frac{\partial \mathbf{u}_t}{\partial \mathbf{n}}\right|_{wall} \quad (13)$$

where $\boldsymbol{\tau}_W$ is wall shear stress, $\mathbf{u}_t$ is tangential velocity and $\mathbf{n}$ is the direction of a unit vector normal to the wall at the moment.

3. CASE STUDY: INFLUENCE OF FINITE ELEMENT MESH QUALITY USED FOR SIMULATING BLOOD FLOW

Simulation of blood flow through the carotid artery in human body was carried out on a realistic three-dimensional geometry in two cases, two different topologies of blocks.

3.1. Block's topologies

The geometry of the artery is obtained by images from CT scanner using STL and CAD programs. A schematic model of the carotid bifurcation is shown in Fig. 1.

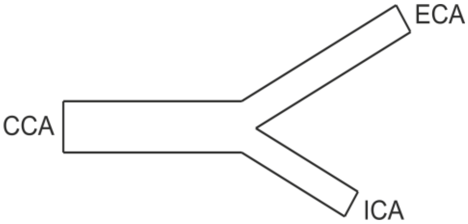
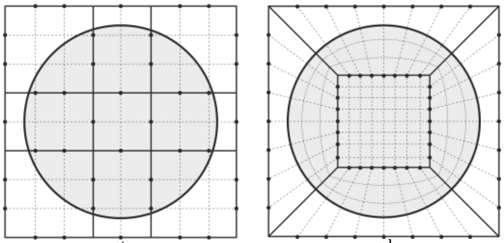


Figure 1. Schematic model of carotid artery bifurcation

The finite element mesh was obtained from discretizing the domain with eight-noded isoparametric solid elements using a multiblock meshing technique. Multiblock method provides different solutions - meshes for different blocks topologies. Mesh is created by own software stl2mesh that has the following tasks: semi-automatic, user assisted, block topology development, and generating eight-noded finite elements inside each of blocks projecting proprietary nodes directly to volumetric mesh. In this way, time-consuming surface reconstruction process is being bypassed, since our ultimate goal is quality finite element meshes.

First case of finite element mesh is created using topology of blocks as on Fig. 2a. In this case deformed unfavorable elements are obtained as a result of the blocks that are closest to the external vertices.

To generate the proper elements of the entire domain, it is necessary to apply the layout of the blocks showed on Fig. 2b. The finite element model developed from this topology has a regular mesh as well as a good representation of its geometry, which would give a better prediction of its biomechanical response.



(a) 1st case (b) 2nd case

Figure 2. Topologies of blocks

On the resulting mesh was applied loads and boundary conditions and in proper format is exported for simulating in software PAK-F Explicit.

3.2. Loads and boundary conditions

Time function that was used is a standard phase of systole and diastole of one cardiac cycle in human body (Fig. 3).

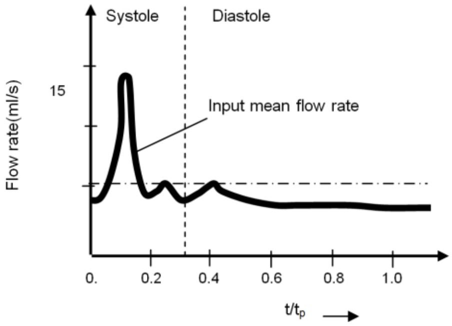


Figure 3. Input flow waveform for one pulse cycle

The boundary conditions for the calculation model are:

- inlet velocity profile is parabolic 3d inlet like a fluid flow through a circular tube,
- on the walls of the artery fluid velocity is set to zero (no-slip condition) and
- on the output side of artery surface forces are set to zero.

3.3. Results of simulation in software PAK-F Explicit

The calculation was performed in 30 steps (10 by 0.02s and 20 by 0.03s which gives in total 0.8s). According to the literature [10-13] following input data is used: the average flow velocity in the inlet $v_{mean} = 16.9\left[cm/s\right]$, density of blood is $\rho = 1050\left[kg/m^3\right]$ and coefficient of dynamic viscosity is $\mu = 0.003675\left[Pa \cdot s\right]$. Results obtained by PAK-F Explicit are printed in *. vtk file as described in [14]. Figure 4. shows the results of wall shear stress for 1st and 2nd case of finite element mesh.

On the external carotid artery (ECA) where cross section is bigger and flow velocity is smaller there is a low value of wall shear stress. In these areas where wall shear stress have small values there is possibility for the occurrence of atherosclerosis.

From the results obtained by analyzing two models of the carotid artery bifurcation it can be seen that the quality of finite element meshes affecting a lot on wall shear stress field. For the 1st case of finite element mesh wall shear stress has bad results that are displayed as a line that runs along the entire artery. In the 2nd case of finite element mesh results are good and show that the wall shear stress have maximum at constriction of blood vessel.

Constriction problems of blood vessel can be successfully solved by installing the stents where there is a possibility of total congestion of blood vessels. After placing the stent, blood vessel lumen and cross section is increased. Then same analysis can be done by following procedure to gain insights into improving the functioning of blood vessels of the patient.

<table>
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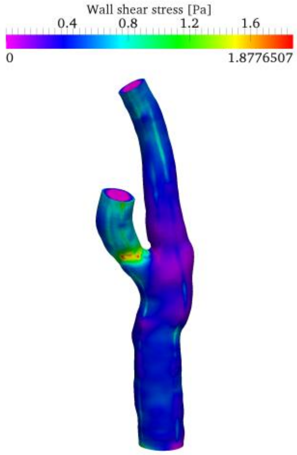
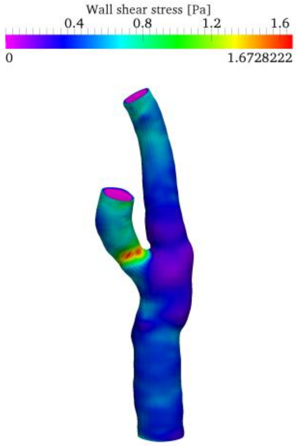
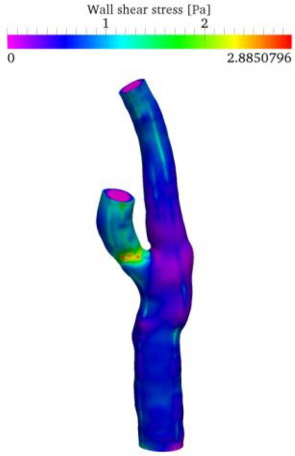
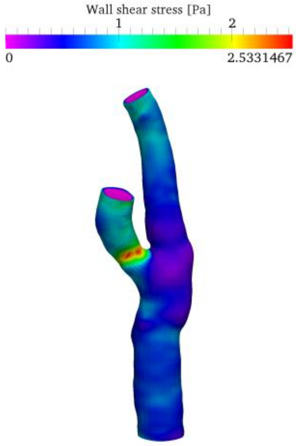
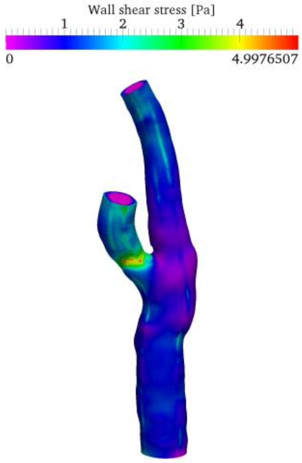
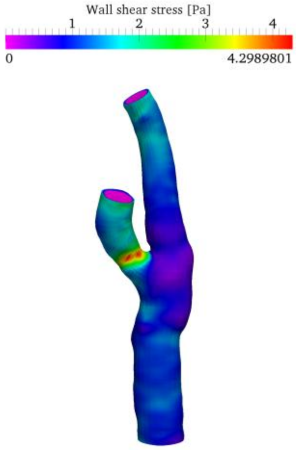
<tr><td>01</td><td></td><td></td></tr>
<tr><td>03</td><td></td><td></td></tr>
<tr><td>05</td><td></td><td></td></tr>
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Figure 4. Wall shear stress

4. CONCLUSION

Provided case study illustrates the application of PAK-F Explicit in the study of hemodynamic characteristics of carotid artery bifurcation. Also, new software PAK–F Explicit is tested and its performances are measured. Software is able to solve the problems of laminar viscous incompressible fluid flow and can be upgraded to solve problems of turbulent fluid flow. In this paper it is described how mesh of finite elements is created and how much this had influence on the final results of blood flow. In future studies it would be useful to compare how quality of the mesh affects on the results of velocity and pressure of blood in the blood vessel.

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