Danijela Tadic[1)]
Jovana Kostic[1)]
Marija Zahar Djordjevic[1)]
Hrvoje Puskaric[1)]

1) Faculty of Engineering, University of Kragujevac, Serbia,
galovic@kg.ac.rs
jovi1119@hotmail.com
maja_199@yahoo.com
xpboje@gmail.com

THE PLANT WASTE MANAGEMENT PROBLEM IN UNCERTAIN ENVIRONMENT

***Abstract**: In this paper, consider is the problem of waste plant management in the presence of uncertain demand and uncertain unit of processing plant raw materials. Uncertainties are modeled by triangular fuzzy numbers. The considered problem is set up as a task stage LP with fuzzy constraints and fuzzy coefficients vectors restrictions. The finding of the optimal solution is based on concept of equal possibilities. The proposed fuzzy LP model is illustrated by example with real-life data.*
***Keywords:** plant waste management, fuzzy sets, fuzzy linear programming, concept of equal possibilities.*

1. INTRODUCTION

The problem of waste plant management in recent decades has become an increasingly topical issue of research in environmental, economic, quality management domain of reverse logistic chains. The term waste plant is defined as waste generated in various processes of processing plants and contains no other ingredients except plants.

In developed countries, the plant is almost no waste stored in nature because it leads to a number of environmental and economic problems. The solution to this problem is based on the ideas that the plant waste to make useful. One of the most common ways is a plant waste processing in compost which is the raw material for the production of many products such as, for example fertilizer for land used in agriculture. In the U.S. there is the council for compost. One of the tasks of this council is to propagate the composting process as essential to economic development and environmental protection. Unfortunately, in Serbia there is no organized production of compost.

This paper considers the problem of determining the optimal amount of plant debris moving in reverse logistics chain so that the objective is to maximize the profit derived from the sale of recycling. The considered problem can formally be set the task of linear programming (LP). In literature can be found in a number of papers which solves a similar problem [1-2].

It should be noted that a number of variables that exist in the problem of waste management plant is not possible adequately describe deterministic values. Inability to express the values of some variables precise numbers is due to poor and / or incomplete records, rapid and frequent changes that occur in the environment, etc.. The development of some areas of mathematics such as the theory of fuzzy sets [3-4], enabled the uncertainty and imprecision quantitatively describe adequately. By applying fuzzy approach in modeling of all uncertainties are eliminated and imprecise. The literature can be found in the papers which consider the problem of supply chain management in the presence of uncertainty

[5-6].

Contributions of this paper are the following: (1) it proposes a fuzzy LP model for reverse logistic chains managementmulti, and (2) it handles uncertainty in: time processing of raw materials to the waste collection point to establish a stable demand for raw materials and demand for the products of the recycling process by using fuzzy sets.

This paper is organized as follows Section 2 data is a problem setting in Section 3 shows the fuzzy model of linear programming (FLP) and go 4 provides an illustrative example with real data and Section 5 presents the conclusions.

2. PROBLEM STATEMENT

Solving the problem of plant waste management affects the quality of life for all residents as well as the profitability of many businesses.

Reverse logistics chain of bio-waste consists of the following entities: (1) the collection of bio-waste, (2) recycling center, and (3) buyers of products made in the recycling process.

Collection point formally by a set of indices I = {1 .. i ... I}, where the index of the collection, and I is the total number of collection. As this paper deals with the collection of plants waste all facilities that manufacture or processing of herbal products. At each collection point i, i = 1, ..., and deliver a number of different plant materials that are formally set K = {1 .. k ... k}. Index of plant material to the total number of plant material was identified as K. It should be noted the amount of each type k, k = 1, .., K, which is collected at the site i, i = 1, ..., I depend on the demand observed in the collection. The values of herbal raw materials demand were based on data from the records, given current market positions of competing companies, etc.. Sideration values determined management team of each collection. Plant waste generated at the site i, i = 1, .., equal to the sum of the waste generated in the processing of vegetable raw material k, k = 1, .., K. Plant waste generated due to the raw material k, k = 1,., K at the site i, i = 1, .., and depends on the percentage of waste and waste of time. Values $a_k, t_{ik}, i = 1,..,I; k = 1,..,K$ depend primarily on the technological level processing of raw materials at the site i, i = 1, I. .. In this paper, introduced the assumption that the values $a_k, k = 1,..,K$ are crisps and uncertain values $t_{ik}, i = 1,..,I; k = 1,..,K$ are described. Plant waste generated at the site and stored, i = 1, ..., I stored in containers and transported to a recycling center.

Recycling centers are formally represent a set J = {1 .. j ... J}, where j is the index for recycling cenar and J is the total number of recycling centers. At each recycling center j, j = 1, .., J applying appropriate technologies for plant waste recycling which caused semi-products can be further processed and finished products sold to customers or stored. Based on data from the literature can be said that less than 10% of the total product recycling the final products [7]. Accordingly, the final product is given no consideration. Semi-product is the set R = {1 .. r ... R}. Total intermediate was identified as R. The index of a semi-products is r, r = 1, .., r. The number and type of semi-productsarising recycling center j, j = 1, ..., J depends on the technological level of recycling center j, j = 1, ..., J, and demand for each type of semi- products r, r = 1, .., R . which is determined primarily on the basis of data from the records, evaluation and knowledge management team reverse logistics chain. The most important types of semi-products is compost.

3. FUZZY LINEAR PROGRAMMING MODEL

In this paper the problem of observed plant waste management throughout the chain of reverse logistics. The model contains a large number of variables. The values of the uncertain demand for a variety of products that move through the reverse logistic chain and the value of time processing plant raw materials are described by management team. Management team provides interval estimates of the value of these parameters. The all uncertain parameters are modeled by triangular fuzz numbers. Triangular function is less tolerant because the range of maximum membership has to be around the crisp point of the triangle. It can be mentioned that the shape of the membership functions can be obtained based on one's experience, subjective belief of decision makers, intuition and contextual knowledge about the concept modeled [4] and should reflect the knowledge and uncertainty aviable on the treated linguistic variables. Fuzzy sets of higher types and levels have not as yet played a significiant role in applications of fuzzy sets theory [8].

In order to facilitate understanding of the proposed FLP model presented in this paper provides further description of the parameters and variables that exist in the proposed model.

x_{ik} -amount of product k, k = 1, .., K, which is delivered at the relevant time and the place of collection, i = 1, .., I.

$\tilde{t}_{ik}$ - - Unit processing time the product k, k = 1, .., K at the site collection, i = 1, .., I, it is assumed that values of parameters depend on this level of technological processes pread plant material, so that it is Modeled by triangular fuzzy numbers (L_{ik}, M_{ik}, U_{ik}) L_{ik}, U_{ik} and the lower, modal and upper limit value of triangular fuzzy number, these values $\tilde{t}_{ik}, i = 1,..,I; k = 1,..,K$ are expressed in minutes.

Q_i - available capacity of the collection and, i = 1, .., I; usually counts as disposable shave hours depending on the number of workers at the site i, i = 1, .., I

$\tilde{d}_{ik}$ -Ask for product k, k = 1, .., K at the site collection, i = 1, .., I. This value is described by fuzzy numbers (l_{ik}, m_{ik}, u_{ik}). Lower, upper limit of the modal value of triangular fuzzy number $\tilde{d}_{ik}$ are labeled l_{ik}, u_{ik}, and m_{ik}, respectively. The values in the domain of the uncertain demand are determined on the basis of data from the records.

a_k the percentage of plant waste generated in the processing unit of product k, k = 1, .., K, this value is determined empirically

b_i -available capacity for storage of waste at the plant collection, i = 1, .., I; usually calculated as the product of the volume of containers and number of containers in the collection and, i = 1, .., I

m_{ij} -percentage value of plant waste that is transported from the collection of i, i = 1, .., and the recycling of j, j = 1, .., J

b_r - Percentage of plant waste to semi-r, r = 1, .., R

c_r - Unit profit semi-product-type r, r = 1, .., R

$\tilde{d}_r$ - Demand for intermediate r, r = 1,., R. This value is described by triangular fuzzy numbers (l_r, m_r, u_r). Lower, modal and upper limit l_r, u_r m_r value in the domain of triangular fuzzy number $\tilde{d}_r$ is determined management team reverse logistics chain.

The formal setting of the task given to the management of plant waste further:

The objective function:

$$\max \sum_{r=1}^{R} c_r b_r \sum_{j=1}^{J} \sum_{i=}^{I} \sum_{k=1}^{K} m_{ij} a_k x_{ik}$$

Limitations:

$$x_{ik} \le \tilde{d}_{ik}, \ i=1,..,I;\ k=1,..,K \quad (1)$$

$$\sum_{k=1}^{K} \tilde{t}_{ik} \cdot x_{ik} \le Q_i \quad (2)$$

$$\sum_{k=1}^{K} a_k \cdot x_{ik} \le b_i \quad (3)$$

$$\sum_{r=1}^{R} b_r \sum_{j=1}^{J} \sum_{i=1}^{I} m_{ij} \sum_{k=1}^{K} a_k x_{ik} \le \tilde{d}_r \quad (4)$$

$$x_{ik} \ge 0$$

The considered problem is stated as fuzzy LP task right side with fuzzy values and fuzzy coefficients of constrains. A simpler version of this problem can be solved by applying the procedures in [9]. In this paper, the optimal solution is obtained by applying the concept of equal opportunity [10].

The concept of equal possibilities was delevloped according to the assumption that membership functions are discretizated. The discretization step is 0.25. For each membership value, LP task is stated. The conventional solution of each LP task is obtained by using LINDO. In this way, the obtained results are fuzzy numbers. It has to be defuzzified. In this paper, defuzzification is performed by using moment method [4].

4. ILLUSTRATIVE EXAMPLE

FLP developed model has been tested on real data. Reverse logistics chain consists of a collection of two (I = 2) and two of waste recycling plant (J = 2). At each collection point, i = 1.2 are delivered to three different types of products, k = 1,2,3. Unit is all three products kilogram. Products of the recycling process in both the compost recycling center (R = 1). Unit profits pounds of compost is 25 dinars. The time period in which management is considered one month.

For each membership function values sets the conventional LP model. The optimal solution of LP problem is calculated by using LINDO software.

It should be noted the following labels: $x_1 = x_{11}, x_2 = x_{12}, x_3 = x_{13}, x_4 = x_{21}, x_5 = x_{22}, x_6 = x_{23}$

The objective function:

$$\max\{5x_1 + x_2 + x_3 + 6x_4 + 0.15x_5 + 2x_6\}$$

Limitations:

(1):

$x_1 \le (400,500,600)$, $x_2 \le (200,250,300)$

$x_3 \le (100,100,120)$,

$x_4 \le (800,1000,1000)$ $x_5 \le (350,400,450)$,

$x_6 \le (180,200,220)$

(2):

$$\tilde{0.16}\,x_1 + \tilde{0.25}\,x_2 + \tilde{0.5}\,x_3 \le 504$$

$$0.2x_4 + 0.3x_5 + 0.56x_6 \le 336$$

So that:

$$\tilde{0.16} = (0.12, 0.16, 0.2)$$

$$\tilde{0.25} = (0.21,\ 0.25,\ 0.29)$$

$$\tilde{0.5} = (0.46, 0.5, 0.54)$$

(3):

$$0.25x_1 + 0.05x_2 + 0.05x_3 \le 1800$$

$$0.3x_4 + 0.08x_5 + 0.1x_6 \le 900$$

(4):

$$0.225x_1 + 0.045x_2 + 0.045x_3 + 0.24x_4 + 0.072x_5 + 0.09x_6 \le (4000,5000,6000)$$

Table 1 shows the optimal values of the fuzzy LP task set for each value of membership function by using LINDO.

Table 1. The optimal values of quantaties in reverse logistic chain and profit values

	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	$f\left(x^*\right)$
$\alpha = 0$	400 600	150 350	60 140	700 900	300 0	100 278.7	6655 9447.14
$\alpha = 0.25$	425 575	175 325	70 130	725 875	325 23.3	125 275	7018.75 9133.5
$\alpha = 0.5$	450 550	200 300	80 120	750 850	340 86.7	150 250	7381 8783
$\alpha = 0.75$	475 525	225 275	90 110	775 825	276.6 150	200 225	7731.5 8432.5
$\alpha = 1$	500	250	100	800	213.3	225	8082

The representative scalars of fuzzy numbers are given by applying moment method (Zimmermann, 2001, Klir, Folger, 1988), so that:

$X_1^* = 505.25$, $X_2^* = 250$, $X_3^* = 68.75$,

$X_4^* = 800$, $X_5^* = 209.68$, $X_6^* = 207.81$,

$f^* = 8081.25$

5. CONCLUSION

In this paper, the problem of waste management plant in the presence of uncertain demand is described as a task stage LP. Demand values are defined on the basis of the records and they are Modeled by triangular fuzzy numbers. The objective function is defined as the total benefits from the sale of compost.

The developed model is flexible in terms of changing the number of entities of reverse logistic chain, as well as the changes in the demand and raw material per unit of time. For each level of membership function, the considered problem is stated as conventional LP task.

The optimal solutions and objective function values are calculated by using LINDO software. By using the concept of equal possibilities, the results obtained are described by discrete fuzzy numbers. The representative of scalars variables and objective function value are given by using defuzzification procedure.

In future work, the proposed fuzzy model can be extended by introducing new Variable costs such as transportation and that such a model tested in large-scale problems.

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